10

INTERACTIONS AND POTENTIAL ENERGY

Conceptual Questions

10.1. Kinetic energy depends on speed. Potential energy depends on position.

10.2. No, kinetic energy can never be negative. Kinetic energy is energy of motion. Motion may stop, but it can't be negative. Speed has no direction and cannot be negative. Yes, gravitational potential energy can be negative. Potential energy depends upon position, which can be positive or negative.

10.3. Conservation of energy tells us that $U_\text{i} = K_\text{f}$, since the car starts at rest. Originally, this means that in rolling down a track of height *h*,

$$mgh = \frac{1}{2}mv_0^2$$

To go twice as fast at the bottom, we must find the height h' such that

$$mgh' = \frac{1}{2}m(2v_0)^2$$

$$\Rightarrow mgh' = 4\left(\frac{1}{2}mv_0^2\right).$$

So $h' = 4h.$ You must increase the track height by a factor of 4.

10.4. $v_\text{a} = v_\text{b} = v_\text{c}$. They each start with the same kinetic energy and they each have the same change in potential energy, so they end with the same kinetic energy and, thus, the same speed.

10.5. $(U_\text{s})_\text{d} > (U_\text{s})_\text{c} > (U_\text{s})_\text{b} = (U_\text{s})_\text{a}$. $U_\text{s} = \frac{1}{2}k(\Delta s)^2$. Increasing the stretch by a factor of 2 increases the stored energy by a factor of 4. Doubling *k* doubles the stored energy.

10.6. The original spring stores energy $U = \frac{1}{2}k(1.0\text{ cm})^2$. For a spring with spring constant $k' = 2k$,

$$U' = \frac{1}{2}k'(\Delta s)^2 = \frac{1}{2}(2k)(\Delta s)^2$$

If $U' = U$, then

$$\frac{1}{2}k(1.0\text{ cm})^2 = \frac{1}{2}(2k)(\Delta s)^2$$

$$\Rightarrow \Delta s = \frac{1}{\sqrt{2}}\text{ cm} = 0.71\text{ cm}$$

10.7. Energy conservation tells us that the initial potential energy stored in the spring is equal to the final kinetic energy of the ball.

$$\frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_0^2$$

When the spring is compressed twice as far,

$$\frac{1}{2}k(2\Delta s)^2 = \frac{1}{2}m(2v_0)^2$$

So the ball speed increases by a factor of 2.

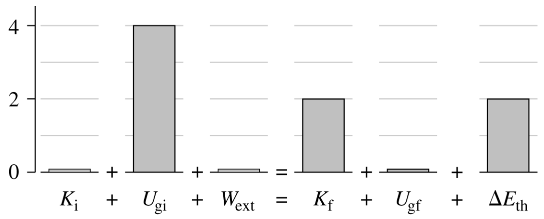
10.8. **(a)** At $x = 6$ m the particle has the most kinetic energy. The kinetic energy is the difference between the total energy (TE) and potential energy (PE) curves. At $x = 3$ m the particle's speed is locally a maximum, but is not as fast as at $x = 6$ m.
(b) The turning points for the particle with total energy (TE) shown are at $x = 2$ m and $x = 8$ m.
(c) The particle could remain at rest in stable equilibrium at $x = 3$ m and $x = 6$ m. The particle could also remain at rest in unstable equilibrium at $x = 1$ m and $x = 4$ m.

10.9. Assume the air resistance is negligible. Choose to include the bock, the spring, the incline, and the earth in the system; then the interactions are within the system.

10.10. There is not enough information to tell. The lost potential energy and the work done by the environment could increase the kinetic energy or it is possible that all the work and energy are converted to thermal energy.

10.11. There is not enough information to tell. The work done could cause some or all of the potential energy change or some of the work could be converted to thermal energy. Without more information, it is impossible to say whether a kinetic energy change is present.

10.12. If we include the pole and the earth in the system then there is no W_{ext}. There will be no gravitational potential energy at the bottom, but there will be some kinetic energy. The bars on the right have to add up the same amount as the bars on the left. So W_{ext} and U_{f} will be zero while K_{f} will be two blocks high.



10.13. **(a)** No, the rate of change of potential energy with respect to position will be zero at that point, but the value of the potential energy is not known without specifying it at some reference point.
(b) No, the zero point for the potential energy is arbitrary. There will be a force present if the rate of change of the potential energy with position is nonzero.

Exercises and Problems

Exercises

Section 10.1 Potential Energy

10.1. **Solve:** **(a)** If A, B, and C are separate systems, then we'll add the ΔK for each system to get the total. If A is stationary then its K doesn't change, so $\Delta K_{\text{A}} = 0$. Because of the work–kinetic energy theorem, the force of A on B increases K_{B} by 10 J and the forces from the environment increase it by 4 J, so $\Delta K_{\text{B}} = 14$ J. The force of A on C decreases its kinetic energy by 5 J and the forces from the environment increase it by 8 J, so $\Delta K_C = 3$ J. The sum of these is $\Delta K_{tot} = \Delta K_A + \Delta K_B + \Delta K_C = 0\text{ J} + 14\text{ J} + 3\text{ J} = 17$ J. With no interactions in the system $\Delta U_{int} = 0$ since the forces are all external to the system.

(b) When we consider all three objects to be one system there is an increase in interaction energy due to Object A on B and C which totals 10 J – 5 J = 5 J. So $\Delta U_{int} = 5$ J. The work done by external forces increases the kinetic energy and is $\Delta K_{tot} = 4\text{ J} + 8\text{ J} = 12$ J.

Assess: Hence the total change in energy is the same regardless of which system we chose.

10.2. Solve: (a) Since $\Delta U_{int} = -W_{int}$, then the work done by interaction forces is 5 J.

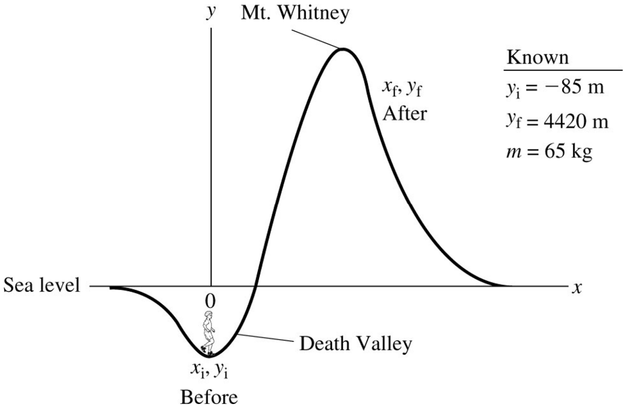
(b) Since $\Delta K_{tot} = W_{ext}$ then the work done by external forces is 7 J.

Assess: Since these don't add up to zero the energy of the system has changed.

Section 10.2 Gravitational Potential Energy

10.3. Model: Model the hiker as a particle.

Visualize:



The origin of the coordinate system chosen for this problem is at sea level so that the hiker's position in Death Valley is $y_0 = -8.5$ m.

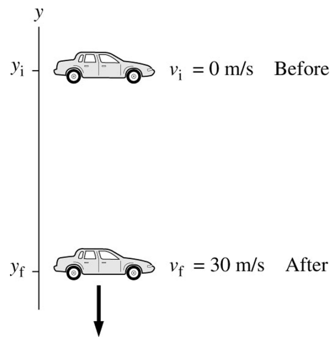
Solve: The hiker's change in potential energy from the bottom of Death Valley to the top of Mt. Whitney is

$$\Delta U = U_{gf} - U_{gi} = mgy_f - mgy_i = mg(y_f - y_i)$$
$$= (65\text{ kg})(9.8\text{ m/s}^2)[4420\text{ m} - (-85\text{ m})] = 2.9\times10^6\text{ J}$$

Assess: Note that ΔU is independent of the origin of the coordinate system.

10.4. Model: Model the car (C) as a particle. This is an example of free fall, and therefore the sum of kinetic and potential energy does not change as the car falls.

Visualize:



Solve: **(a)** The kinetic energy of the car is

$$K_C = \frac{1}{2}m_C v_C^2 = \frac{1}{2}(1500\text{ kg})(30\text{ m/s})^2 = 6.75\times10^5\text{ J}$$

The car's kinetic energy is 6.8×10^5 J.

(b) Let us relabel K_C as K_f and place our coordinate system at $y_f = 0$ m so that the car's potential energy U_{gf} is zero, its velocity is v_f, and its kinetic energy is K_f. At position y_i, $v_i = 0$ m/s or $K_i = 0$ J, and the only energy the car has is $U_{gi} = mgy_i$. Since the sum $K + U_g$ is unchanged by motion, $K_f + U_{gf} = K_i + U_{gi}$. This means

$$K_f + mgy_f = K_i + mgy_i \Rightarrow K_f + 0 = K_i + mgy_i$$

$$\Rightarrow y_i = \frac{(K_f - K_i)}{mg} = \frac{(6.75\times10^5\text{ J} - 0\text{ J})}{(1500\text{ kg})(9.8\text{ m/s}^2)} = 46\text{ m}$$

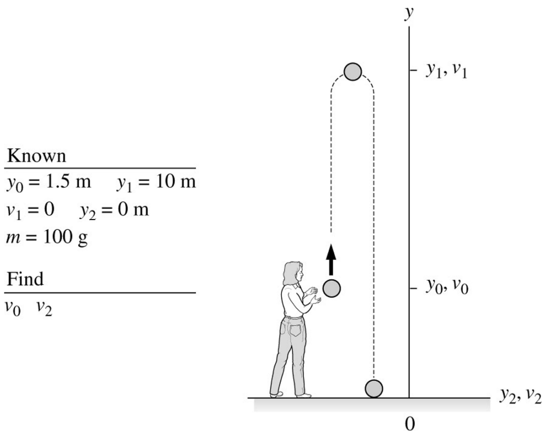
(c) From part **(b)**,

$$y_i = \frac{(K_f - K_i)}{mg} = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{mg} = \frac{(v_f^2 - v_i^2)}{2g}$$

Free fall does *not* depend upon the mass.

10.5. Model: This is a problem of free fall. The sum of the kinetic and gravitational potential energy for the ball, considered as a particle, does not change during its motion.

Visualize:



The figure shows the ball's before-and-after pictorial representation for the two situations described in parts **(a)** and **(b)**.

Solve: The quantity $K + U_g$ is the same during free fall. Thus, $K_f + U_{gf} = K_i + U_{gi}$.

(a) $\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0 \Rightarrow v_0^2 = v_1^2 + 2g(y_1 - y_0)$

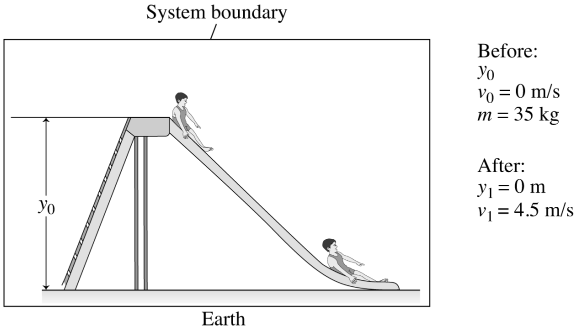
$\Rightarrow v_0^2 = (0\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(10\text{ m} - 1.5\text{ m}) = 166.6\text{ m}^2/\text{s}^2 \Rightarrow v_0 = 12.9\text{ m/s} \approx 13\text{ m/s}$

(b) $\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_0^2 + mgy_0 \Rightarrow v_2^2 = v_0^2 + 2g(y_0 - y_2)$

$\Rightarrow v_2^2 = 166.6\text{ m}^2/\text{s}^2 + 2(9.8\text{ m/s}^2)(1.5\text{ m} - 0\text{ m}) \Rightarrow v_2 = 14\text{ m/s}$

Assess: An increase in speed from 12.9 m/s to 14.0 m/s as the ball falls through a distance of 1.5 m is reasonable. Also, note that mass does not appear in the calculations that involve free fall.

10.6. Model: Model the child as a particle. The slide is frictionless and the child starts from rest. Include the child, the slide, and the earth in the system.
Visualize: Draw a before-and-after pictorial representation.



Solve: There is no external work done on the system.

$$\Delta E_{\text{sys}} = \Delta K + \Delta U_{\text{G}} = W_{\text{ext}} = 0$$

That is, the system energy does not change. Kinetic energy is transformed into potential energy without loss inside the system. In principle, the kinetic energy is that of the ball plus the kinetic energy of the earth, but the enormous mass difference means that the earth effectively remains at rest, so we consider only the kinetic energy of the child.

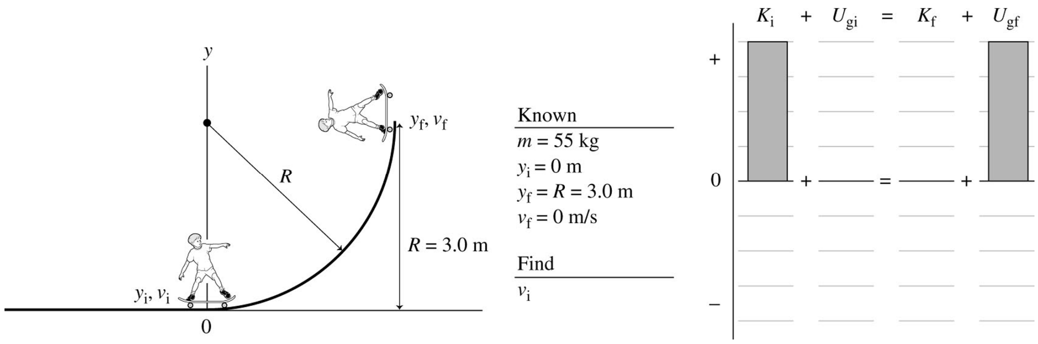
$$0 = \Delta K + \Delta U_{\text{G}} = \left(\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2\right) + (mgy_1 - mgy_0)$$

We know that $v_0 = 0$ m/s and we chose a coordinate system in which $y_1 = 0$ m, so we're left with

$$y_0 = \frac{v_1^2}{2g} = \frac{(4.5\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 1.0\ \text{m}$$

The answer does not depend on the child's mass or the angle of incline of the frictionless slide.
Assess: This is not a very tall slide, but most slides aren't frictionless.

10.7. Model: Model the skateboarder as a particle. Assuming that the track offers no rolling friction, the sum of the skateboarder's kinetic and gravitational potential energy does not change during his rolling motion.
Visualize:



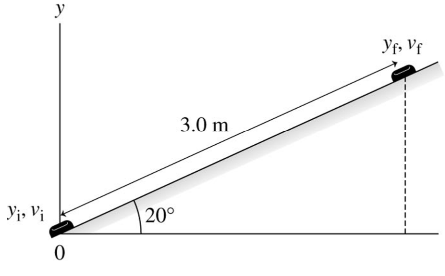
The vertical displacement of the skateboarder is equal to the radius of the track.
Solve: The quantity $K + U_\text{g}$ is the same at the upper edge of the quarter-pipe track as it was at the bottom. The energy conservation equation $K_\text{f} + U_\text{gf} = K_\text{i} + U_\text{gi}$ is

$$\frac{1}{2}mv_\text{f}^2 + mgy_\text{f} = \frac{1}{2}mv_\text{i}^2 + mgy_\text{i} \Rightarrow v_\text{i}^2 = v_\text{f}^2 + 2g(y_\text{f} - y_\text{i})$$

$$v_\text{i}^2 = (0\ \text{m/s})^2 + 2(9.8\ \text{m/s}^2)(3.0\ \text{m} - 0\ \text{m}) = 58.8\ \text{m}^2/\text{s}^2 \Rightarrow v_\text{i} = 7.7\ \text{m/s}$$

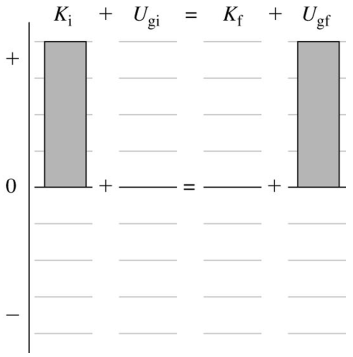
Assess: Note that we did not need to know the skateboarder's mass, as is the case with free-fall motion.

10.8. Model: Model the puck as a particle. Since the ramp is frictionless, the sum of the puck's kinetic and gravitational potential energy does not change during its sliding motion.
Visualize:



Known
$m = 100$ g
$y_i = 0$ m
$y_f = (3.0 \text{ m}) \sin 20° = 1.03$ m
$v_f = 0$ m/s

Find
v_i



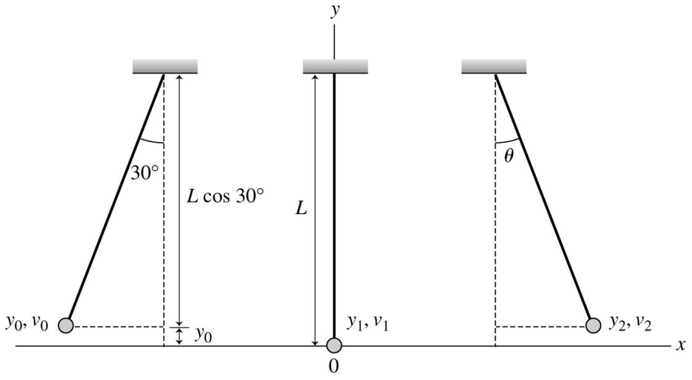
Solve: The quantity $K + U_g$ is the same at the top of the ramp as it was at the bottom. The energy conservation equation $K_f + U_{gf} = K_i + U_{gi}$ is

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i \Rightarrow v_i^2 = v_f^2 + 2g(y_f - y_i)$$

$$\Rightarrow v_i^2 = (0 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(1.03 \text{ m} - 0 \text{ m}) = 20.2 \text{ m}^2/\text{s}^2 \Rightarrow v_i = 4.5 \text{ m/s}$$

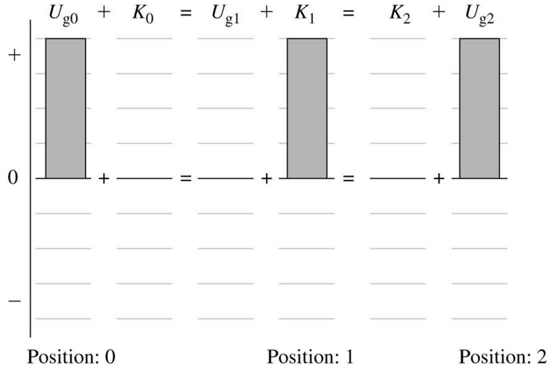
Assess: An initial push with a speed of $4.5 \text{ m/s} \approx 10 \text{ mph}$ to cover a distance of 3.0 m up a $20°$ ramp seems reasonable.

10.9. Model: In the absence of frictional and air-drag effects, the sum of the kinetic and gravitational potential energy does not change as the pendulum swings from one side to the other.
Visualize:



Known
$m = 500$ g
$L = 75$ cm
$y_1 = 0$
$v_0 = 0 = v_2$

Find
v_1 θ



The figure shows the pendulum's before-and-after pictorial representation for the two situations described in parts **(a)** and **(b)**.

Solve: **(a)** The quantity $K+U_\text{g}$ is the same at the lowest point of the trajectory as it was at the highest point. Thus, $K_1+U_{\text{g}1}=K_0+U_{\text{g}0}$ means

$$\frac{1}{2}mv_1^2+mgy_1=\frac{1}{2}mv_0^2+mgy_0 \Rightarrow v_1^2+2gy_1=v_0^2+2gy_0$$
$$\Rightarrow v_1^2+2g(0\text{ m})=(0\text{ m/s})^2+2gy_0 \Rightarrow v_1=\sqrt{2gy_0}$$

From the pictorial representation, we find that $y_0=L-L\cos30°$. Thus,

$$v_1=\sqrt{2gL(1-\cos30°)}=\sqrt{2(9.8\text{ m/s}^2)(0.75\text{ m})(1-\cos30°)}=1.403\text{ m/s}$$

The speed at the lowest point is 1.4 m/s.

(b) Since the quantity $K+U_\text{g}$ does not change, $K_2+U_{g2}=K_1+U_{g1}$. We have

$$\frac{1}{2}mv_2^2+mgy_2=\frac{1}{2}mv_1^2+mgy_1 \Rightarrow y_2=(v_1^2-v_2^2)/2g$$
$$\Rightarrow y_2=[(1.403\text{ m/s})^2-(0\text{ m/s})^2]/(2\times9.8\text{ m/s}^2)=0.100\text{ m}$$

Since $y_2=L-L\cos\theta$, we obtain

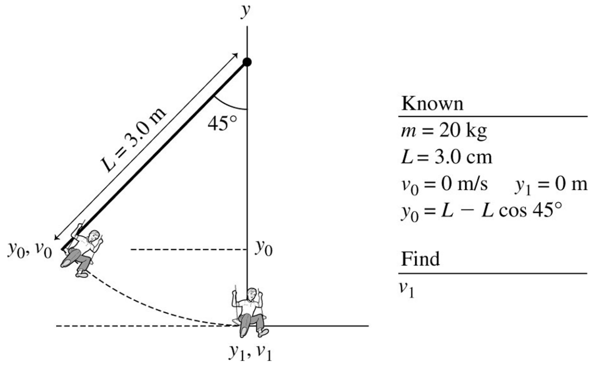
$$\cos\theta=\frac{L-y_2}{L}=\frac{(0.75\text{ m})-(0.10\text{ m})}{(0.75\text{ m})}=0.8667 \Rightarrow \theta=\cos^{-1}(0.8667)=30°$$

That is, the pendulum swings to the other side by 30°.

Assess: The swing angle is the same on either side of the rest position. This result is a consequence of the fact that the sum of the kinetic and gravitational potential energy does not change. This is shown as well in the energy bar chart in the figure.

10.10. **Model:** Model the child and swing as a particle, and assume the chain to be massless. In the absence of frictional and air-drag effects, the sum of the kinetic and gravitational potential energy does not change during the swing's motion.

Visualize:



Solve: The quantity $K+U_\text{g}$ is the same at the highest point of the swing as it is at the lowest point. That is, $K_0+U_{\text{g}0}=K_1+U_{\text{g}1}$. It is clear from this equation that maximum kinetic energy occurs where the gravitational potential energy is the least. This is the case at the lowest position of the swing. At this position, the speed of the swing and child will also be maximum. The above equation is

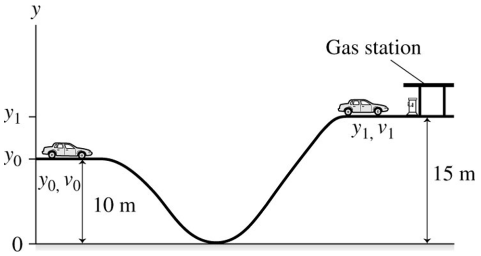
$$\frac{1}{2}mv_0^2+mgy_0=\frac{1}{2}mv_1^2+mgy_1 \Rightarrow v_1^2=v_0^2+2g(y_0-y_1)$$
$$\Rightarrow v_1^2=(0\text{ m/s})^2+2g(y_0-0\text{ m}) \Rightarrow v_1=\sqrt{2gy_0}$$

We see from the pictorial representation that

$$y_0 = L - L\cos 45° = (3.0\text{ m}) - (3.0\text{ m})\cos 45° = 0.879\text{ m}$$
$$\Rightarrow v_1 = \sqrt{2gy_0} = \sqrt{2(9.8\text{ m/s}^2)(0.879\text{ m})} = 4.2\text{ m/s}$$

Assess: We did not need to know the swing's or the child's mass. Also, a maximum speed of 4.2 m/s is reasonable.

10.11. Model: Model the car as a particle with zero rolling friction and no air resistance. The sum of the kinetic and gravitational potential energy, therefore, does not change during the car's motion.
Visualize:



Solve: The initial energy of the car is

$$K_0 + U_{g0} = \frac{1}{2}mv_0^2 + mgy_0 = \frac{1}{2}(1500\text{ kg})(10.0\text{ m/s})^2 + (1500\text{ kg})(9.8\text{ m/s}^2)(10\text{ m}) = 2.22\times10^5\text{ J}$$

The car increases its height to 15 m at the gas station. The conservation of energy equation $K_0 + U_{g0} = K_1 + U_{g1}$ is

$$2.22\times10^5\text{ J} = \frac{1}{2}mv_1^2 + mgy_1 \Rightarrow 2.22\times10^5\text{ J} = \frac{1}{2}(1500\text{ kg})v_1^2 + (1500\text{ kg})(9.8\text{ m/s}^2)(15\text{ m})$$
$$\Rightarrow v_1 = 1.4\text{ m/s}$$

Assess: A lower speed at the gas station is reasonable because the car has decreased its kinetic energy and increased its potential energy compared to its starting values.

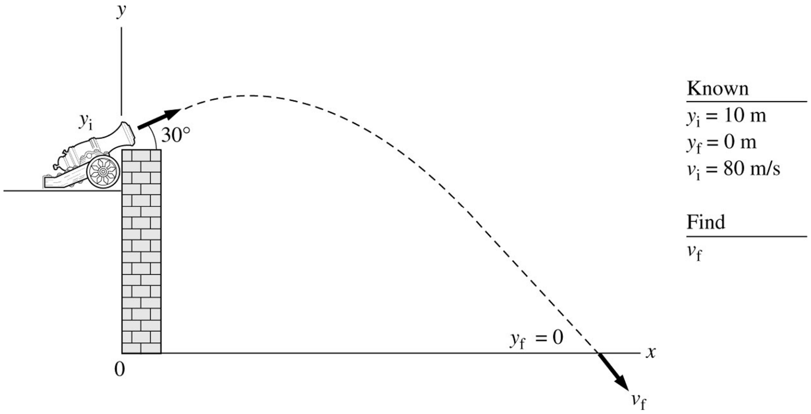
10.12. Model: We will take the system to be the person plus the earth.
Visualize: When a person drops from a certain height, the initial potential energy is transformed to kinetic energy. When the person hits the ground, if they land rigidly upright, we assume that all of this energy is transformed into elastic potential energy of the compressed leg bones. The maximum energy that can be absorbed by the leg bones is 200 J; this limits the maximum height.
Solve: (a) The initial potential energy can be at most 200 J, so the height h of the jump is limited by $mgh = 200$ J. For $m = 60$ kg, this limits the height to

$$h = 200\text{ J/mg} = 200\text{ J/(60 kg)(9.8 m/s}^2) = 0.34\text{ m}$$

(b) If some of the energy is transformed to other forms than elastic energy in the bones, the initial height can be greater. If a person flexes her legs on landing, some energy is transformed to thermal energy. This allows for a greater initial height.
Assess: There are other tissues in the body with elastic properties that will absorb energy as well, so this limit is quite conservative.

10.13. Model: This is case of free fall, so the sum of the kinetic and gravitational potential energy does not change as the cannon ball falls.

Visualize:



The figure shows a before-and-after pictorial representation. To express the gravitational potential energy, we put the origin of our coordinate system on the ground below the fortress.

Solve: Using $y_f = 0$ and the equation $K_i + U_{gi} = K_f + U_{gf}$ we get

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f \Rightarrow v_i^2 + 2gy_i = v_f^2$$

$$v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{(80\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(10\text{ m})} = 81\text{ m/s}$$

Assess: Note that we did not need to use the tilt angle of the cannon, because kinetic energy is a scalar. Also note that using the energy conservation equation, we can find only the magnitude of the final velocity, not the direction of the velocity vector.

10.14. Solve: (a) The change in the potential energy of 1.0 kg of water in falling 25 m is

$$\Delta U_g = -mgh = -(1.0\text{ kg})(9.8\text{ m/s}^2)(25\text{ m}) = -245\text{ J} \approx -0.25\text{ kJ}$$

(b) The power required of the dam is

$$P = \frac{W}{t} = \frac{W}{1\text{ s}} = 50\times10^6\text{ Watts} \quad\Rightarrow\quad W = 50\times10^6\text{ J}$$

That is, 50×10^6 J of energy is required per second for the dam. Out of the 245 J of lost potential energy, $(245\text{ J})(0.80) = 196\text{ J}$ is converted to electrical energy. Thus, the amount of water needed per second is $(50\times10^6\text{ J})(1.0\text{ kg/196 J}) = 255{,}000\text{ kg} \approx 2.6\times10^5\text{ kg}$.

Section 10.3 Elastic Potential Energy

10.15. Model: Assume an ideal spring that obeys Hooke's law.

Solve: The elastic potential energy of a spring is defined as $U_s = \frac{1}{2}k(\Delta s)^2$, where Δs is the magnitude of the stretching or compression relative to the unstretched or uncompressed length. $\Delta U_s = 0$ when the spring is at its equilibrium length and $\Delta s = 0$. We have $U_s = 200$ J and $k = 1000$ N/m. Solving for Δs:

$$\Delta s = \sqrt{2U_s/k} = \sqrt{2(200\text{ J})/1000\text{ N/m}} = 0.632\text{ m}$$

10.16. Model: Assume the spring is ideal and obeys Hooke's law. Then the potential energy of a stretched spring is $U_{\text{sp}} = \frac{1}{2}k(\Delta s)^2$.

Visualize: Use ratios to solve this problem. Use primed variables for the new situation with the spring stretched three times as far.

Solve:

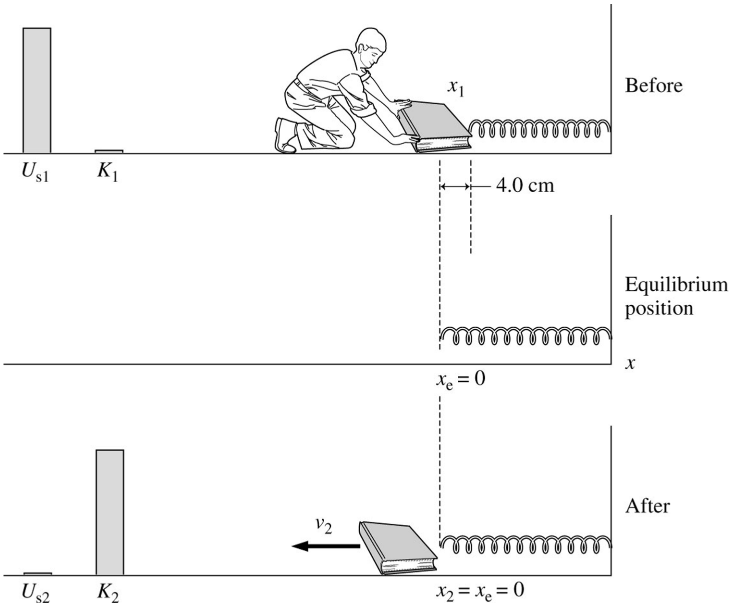
$$\frac{U'_{\text{sp}}}{U_{\text{sp}}} = \frac{\frac{1}{2}k(\Delta s')^2}{\frac{1}{2}k(\Delta s)^2} = \frac{\frac{1}{2}k(3\Delta s)^2}{\frac{1}{2}k(\Delta s)^2} = 3^2 = 9$$

$$U'_{\text{sp}} = 9U_{\text{sp}} = 9(2.0\text{ J}) = 18\text{ J}$$

Assess: The stored energy scales with the square of the spring stretch.

10.17. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, so the mechanical energy $K + U_{\text{s}}$ is conserved. Also model the book as a particle.

Visualize:



The figure shows a before-and-after pictorial representation. The compressed spring will push on the book until the spring has returned to its equilibrium length. We put the origin of our coordinate system at the equilibrium position of the free end of the spring. The energy bar chart shows that the potential energy of the compressed spring is entirely transformed into the kinetic energy of the book.

Solve: The conservation of energy equation $K_2 + U_{\text{s2}} = K_1 + U_{\text{s1}}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_{\text{e}})^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_{\text{e}})^2$$

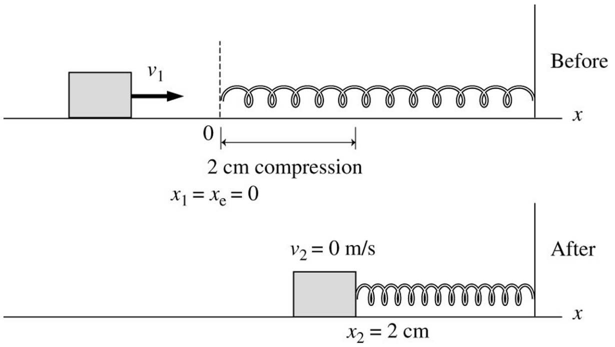
Using $x_2 = x_{\text{e}} = 0$ m and $v_1 = 0$ m/s, this simplifies to

$$\frac{1}{2}mv_2^2 = \frac{1}{2}k(x_1 - 0\text{ m})^2 \Rightarrow v_2 = \sqrt{\frac{kx_1^2}{m}} = \sqrt{\frac{(1250\text{ N/m})(0.040\text{ m})^2}{(0.500\text{ kg})}} = 2.0\text{ m/s}$$

Assess: This problem cannot be solved using constant-acceleration kinematic equations. The acceleration is not a constant in this problem, since the spring force, given as $F_{\text{s}} = -k\Delta x$, is directly proportional to Δx or $|x - x_{\text{e}}|$.

10.18. Model: Assume an ideal spring that obeys Hooke's law. Since there is no friction, the mechanical energy $K+U_s$ is conserved. Also, model the block as a particle.

Visualize:



The figure shows a before-and-after pictorial representation. We have put the origin of our coordinate system at the equilibrium position of the free end of the spring. This gives us $x_1 = x_e = 0$ cm and $x_2 = 2.0$ cm.

Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

Using $v_2 = 0$ m/s, $x_1 = x_e = 0$ m, and $x_2 - x_e = 0.020$ m, we get

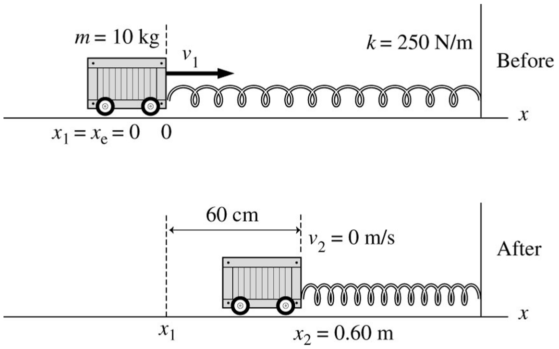
$$\frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 \Rightarrow \Delta x = (x_2 - x_e) = \sqrt{\frac{m}{k}}v_1$$

That is, the compression is directly proportional to the velocity v_1. When the block collides with the spring with twice the earlier velocity $(2v_1)$, the compression will also be doubled to $2(x_2 - x_e) = 2(2.0 \text{ cm}) = 4.0$ cm.

Assess: This problem shows the power of using energy conservation over using Newton's laws in solving problems involving nonconstant acceleration.

10.19. Model: Model the grocery cart as a particle and the spring as an ideal that obeys Hooke's law. We will also assume zero rolling friction during the compression of the spring, so that mechanical energy is conserved.

Visualize:



The figure shows a before-and-after pictorial representation. The "before" situation is when the cart hits the spring in its equilibrium position. We put the origin of our coordinate system at this equilibrium position of the free end of the spring. This gives $x_1 = x_e = 0$ and $(x_2 - x_e) = 60$ cm.

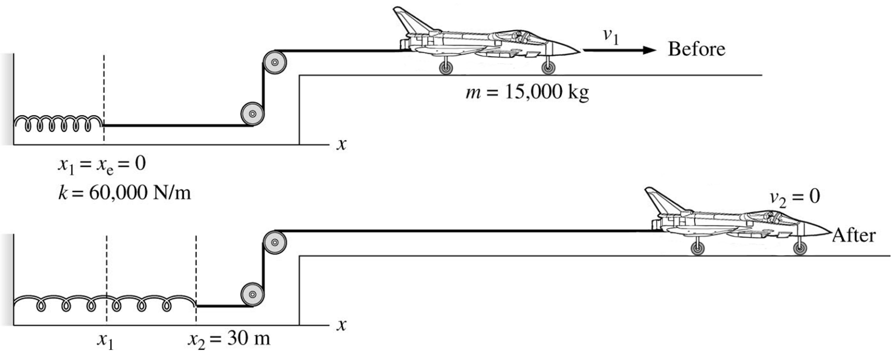
Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

Using $v_2 = 0$ m/s, $(x_2 - x_e) = 0.60$ m, and $x_1 = x_e = 0$ m gives:

$$\frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{\frac{k}{m}}(x_2 - x_e) = \sqrt{\frac{250\ \text{N/m}}{10\ \text{kg}}}(0.60\ \text{m}) = 3.0\ \text{m/s}$$

10.20. Model: Model the jet plane as a particle, and the spring as an ideal that obeys Hooke's law. We will also assume zero rolling friction during the stretching of the spring, so that mechanical energy is conserved.
Visualize:



The figure shows a before-and-after pictorial representation. The "before" situation occurs just as the jet plane lands on the aircraft carrier and the spring is in its equilibrium position. We put the origin of our coordinate system at the right free end of the spring. This gives $x_1 = x_e = 0$ m. Since the spring stretches 30 m to stop the plane, $x_2 - x_e = 30$ m.

Solve: The conservation of energy equation $K_2 + U_{s2} = K_1 + U_{s1}$ for the spring-jet plane system is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(x_1 - x_e)^2$$

Using $v_2 = 0$ m/s, $x_1 = x_e = 0$ m, and $x_2 - x_e = 30$ m yields

$$\frac{1}{2}k(x_2 - x_e)^2 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{\frac{k}{m}}(x_2 - x_1) = \sqrt{\frac{60{,}000\ \text{N/m}}{15{,}000\ \text{kg}}}(30\ \text{m}) = 60\ \text{m/s}$$

Assess: A landing speed of 60 m/s or ≈ 120 mph is reasonable.

10.21. Model: We will assume the knee extensor tendon behaves according to Hooke's law and stretches in a straight line.

Visualize: The elastic energy stored in a spring is given by $U_s = \frac{1}{2}k(\Delta s)^2$.

Solve: For athletes,

$$U_{\text{s,athlete}} = \tfrac{1}{2}k(\Delta s)^2 = \frac{1}{2}(33{,}000\ \text{N/m})(0.041\ \text{m})^2 = 27.7\ \text{J}$$

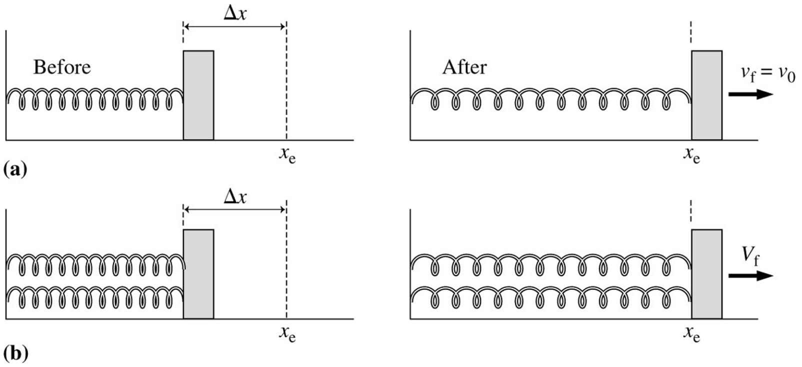
For non-athletes,

$$U_{\text{s,non-athlete}} = \tfrac{1}{2}k(\Delta s)^2 = \frac{1}{2}(33{,}000\ \text{N/m})(0.033\ \text{m})^2 = 18.0\ \text{J}$$

The difference in energy stored between athletes and non-athletes is therefore 9.7 J.
Assess: Notice the energy stored by athletes is over 1.5 times the energy stored by non-athletes.

10.22. Model: Model the block as a particle and the springs as ideal springs obeying Hooke's law. There is no friction, hence the mechanical energy $K + U_s$ is conserved.

Visualize:



Note that $x_f = x_e$ and $x_i - x_e = \Delta x$. The before-and-after pictorial representations show that we put the origin of the coordinate system at the equilibrium position of the free end of the springs.

Solve: The conservation of energy equation $K_f + U_{sf} = K_i + U_{si}$ for the single spring is

$$\frac{1}{2}mv_f^2 + \frac{1}{2}k(x_f - x_e)^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}k(x_i - x_e)^2$$

Using the value for v_f given in the problem, we get

$$\frac{1}{2}mv_0^2 + 0\text{ J} = 0\text{ J} + \frac{1}{2}k(\Delta x)^2 \Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}k(\Delta x)^2$$

Conservation of energy for the two-spring case:

$$\frac{1}{2}mV_f^2 + 0\text{ J} = 0\text{ J} + \frac{1}{2}k(x_i - x_e)^2 + \frac{1}{2}k(x_i - x_e)^2 \quad \frac{1}{2}mV_f^2 = k(\Delta x)^2$$

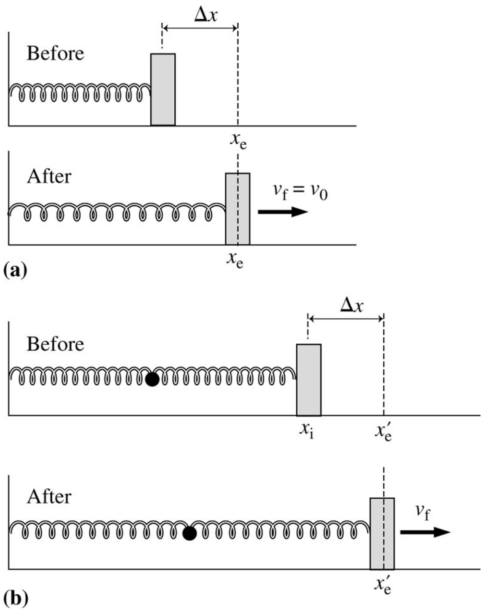
Using the result of the single-spring case,

$$\frac{1}{2}mV_f^2 = mv_0^2 \Rightarrow V_f = \sqrt{2}v_0$$

Assess: The block separates from the spring at the equilibrium position of the spring.

10.23. Model: Model the block as a particle and the springs as ideal springs obeying Hooke's law. There is no friction, hence the mechanical energy $K+U_\text{s}$ is conserved.

Visualize:



The springs in both cases have the same compression Δx. We put the origin of the coordinate system at the equilibrium position of the free end of the spring for the single-spring case **(a)**, and at the free end of the two connected springs for the two-spring case **(b)**.

Solve: The conservation of energy for the single-spring case:

$$K_\text{f}+U_\text{sf}=K_\text{i}+U_\text{si}\Rightarrow\frac{1}{2}mv_\text{f}^2+\frac{1}{2}k(x_\text{f}-x_\text{e})^2=\frac{1}{2}mv_\text{i}^2+\frac{1}{2}k(x_\text{i}-x_\text{e})^2$$

Using $x_\text{f}=x_\text{e}=0$ m, $v_\text{i}=0$ m/s, and $v_\text{f}=v_0$, this equation simplifies to

$$\frac{1}{2}mv_0^2=\frac{1}{2}k(\Delta x)^2$$

Conservation of energy in the case of the two springs in series, where each spring compresses by $\Delta x/2$, is

$$K_\text{f}+U_\text{sf}=K_\text{i}+U_\text{si}\Rightarrow\frac{1}{2}mV_\text{f}^2+0=\frac{1}{2}mv_\text{i}^2+\frac{1}{2}k(\Delta x/2)^2+\frac{1}{2}k(\Delta x/2)^2$$

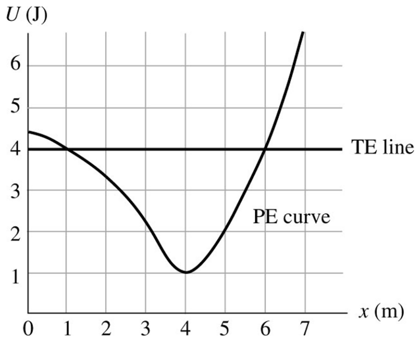
Using $x_\text{f}=x'_\text{e}=0$ m and $v_\text{i}=0$ m/s, we get

$$\frac{1}{2}mV_\text{f}^2=\frac{1}{2}\left[\frac{1}{2}k(\Delta x)^2\right]$$

Comparing the two results we see that $V_\text{f}=v_0/\sqrt{2}$.

Assess: The block pushes on the spring until the spring has returned to its equilibrium length.

10.24. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.
Visualize:



The particle is released from rest at $x = 1.0$ m. That is, $K = 0$ at $x = 1.0$ m. Since the total energy is given by $E = K + U$, we can draw a horizontal total energy (TE) line through the point of intersection of the potential energy curve (PE) and the $x = 1.0$ m line. The distance from the PE curve to the TE line is the particle's kinetic energy. These values are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ does not change.

Solve: (a) We have $E = 4.0$ J and this energy is a constant. For $x < 1.0, U > 4.0$ J and, therefore, K must be negative to keep E the same (note that $K = E - U$ or $K = 4.0\text{ J} - U$). Since negative kinetic energy is unphysical, the particle cannot move to the left. That is, the particle will move to the right of $x = 1.0$ m.

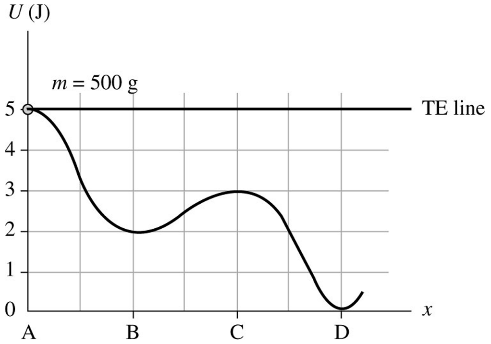
(b) The expression for the kinetic energy is $E - U$. This means the particle has maximum speed or maximum kinetic energy when U is minimum. This happens at $x = 4.0$ m. Thus,

$$K_{\text{max}} = E - U_{\text{min}} = (4.0\text{ J}) - (1.0\text{ J}) = 3.0\text{ J} \qquad \frac{1}{2}mv_{\text{max}}^2 = 3.0\text{ J} \Rightarrow v_{\text{max}} = \sqrt{\frac{2(3.0\text{ J})}{m}} = \sqrt{\frac{8.0\text{ J}}{0.020\text{ kg}}} = 17.3\text{ m/s}$$

The particle possesses this speed at $x = 4.0$ m.

(c) The total energy (TE) line intersects the potential energy (PE) curve at $x = 1.0$ m and $x = 6.0$ m. These are the turning points of the motion.

10.25. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.
Visualize:



The particle with a mass of 500 g is released from rest at A. That is, $K = 0$ at A. Since $E = K + U = 0\text{ J} + U$, we can draw a horizontal TE line through $U = 5.0$ J. The distance from the PE curve to the TE line is the particle's kinetic energy. These values are transformed as the position changes, causing the particle to speed up or slow down, but the sum $K + U$ does not change.

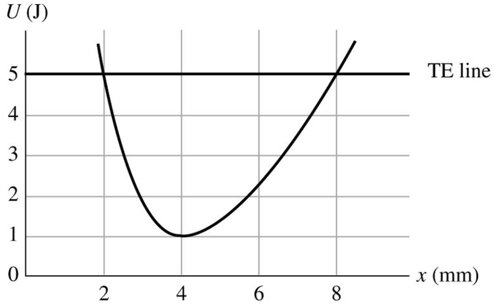
Solve: The kinetic energy is given by $E-U$, so we have

$$\frac{1}{2}mv^2 = E-U \Rightarrow v = \sqrt{2(E-U)/m}$$

Using $U_B = 2.0 \text{ J}$, $U_C = 3.0 \text{ J}$, and $U_D = 0 \text{ J}$, we get

$$v_B = \sqrt{2(5.0 \text{ J} - 2.0 \text{ J})/0.500 \text{ kg}} = 3.5 \text{ m/s} \qquad v_C = \sqrt{2(5.0 \text{ J} - 3.0 \text{ J})/0.500 \text{ kg}} = 2.8 \text{ m/s}$$
$$v_D = \sqrt{2(5.0 \text{ J} - 0 \text{ J})/0.500 \text{ kg}} = 4.5 \text{ m/s}$$

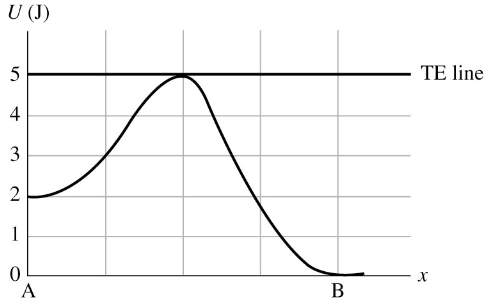
10.26. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.
Visualize:



Since the particle oscillates between $x = 2.0$ mm and $x = 8.0$ mm, the speed of the particle is zero at these points. That is, for these values of x, $E = U = 5.0$ J, which defines the total energy (TE) line. The distance from the potential energy (PE) curve to the TE line is the particle's kinetic energy. These values are transformed as the position changes, but the sum $K+U$ does not change.
Solve: The equation for total energy $E = U + K$ means $K = E - U$, so that K is maximum when U is minimum. We have

$$K_{\text{max}} = \frac{1}{2}mv_{\text{max}}^2 = 5.0 \text{ J} - U_{\text{min}}$$
$$\Rightarrow v_{\text{max}} = \sqrt{2(5.0 \text{ J} - U_{\text{min}})/m} = \sqrt{2(5.0 \text{ J} - 1.0 \text{ J})/0.0020 \text{ kg}} = 63 \text{ m/s}$$

10.27. Model: For an energy diagram, the sum of the kinetic and potential energy is a constant.
Visualize:



For the speed of the particle at A that is needed to reach B to be a minimum, the particle's kinetic energy as it reaches the top must be zero. Similarly, the minimum speed at B for the particle to reach A obtains when the particle just makes it to the top with zero kinetic energy.

Solve: **(a)** The energy equation $K_A + U_A = K_{top} + U_{top}$ is

$$\frac{1}{2}mv_A^2 + U_A = 0\text{ J} + U_{top}$$

$$\Rightarrow v_A = \sqrt{2(U_{top} - U_A)/m} = \sqrt{2(5.0\text{ J} - 2.0\text{ J})/0.100\text{ kg}} = 7.7\text{ m/s}$$

(b) To go from point B to point A, $K_B + U_B = K_{top} + U_{top}$ is

$$\frac{1}{2}mv_B^2 + U_B = 0\text{ J} + U_{top}$$

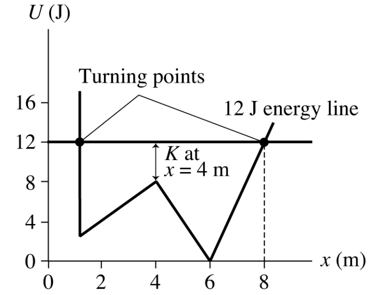
$$\Rightarrow v_B = \sqrt{2(U_{top} - U_B)/m} = \sqrt{2(5.0\text{ J} - 0\text{ J})/0.100\text{ kg}} = 10.0\text{ m/s}$$

Assess: The particle requires a higher kinetic energy to reach A from B than to reach B from A.

Section 10.4 Conservation of Energy

Section 10.5 Energy Diagrams

10.28. Model: Use the model of the conservation of mechanical energy.
Visualize:



Solve: **(a)** The turning points occur where the total energy line crosses the potential energy curve. For $E = 12$ J, this occurs at the points $x = 1$ m and $x = 8$ m.

(b) The equation for kinetic energy $K = E - U$ gives the distance between the potential energy curve and total energy line. $U = 8$ J at $x = 4$ m, so $K = 12\text{ J} - 8\text{ J} = 4$ J. The speed corresponding to this kinetic energy is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(4\text{ J})}{0.5\text{ kg}}} = 4.0\text{ m/s}$$

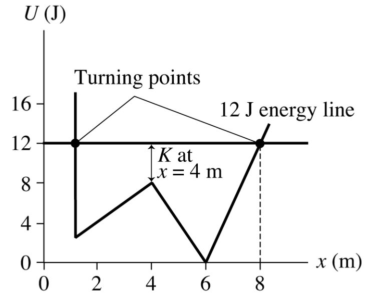
(c) Maximum speed occurs for minimum *U*. This occurs at $x = 6$ m where $U = 0$ J and $K = 12$ J. The speed at this point is

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(12\text{ J})}{0.500\text{ kg}}} = 6.9\text{ m/s}$$

(d) If the particle has $E = 4$ J it be at $x = 2.0$ m, but it would have zero speed; it would be a turning point. The particle could not be at $x = 4.0$ m because that would mean it had more potential energy than total energy.

10.29. Model: Use the model of the conservation of mechanical energy.

Visualize:



Solve: To never reach $x = 6.0$ m it must have zero kinetic energy at the top of the energy mountain at $x = 4.0$ m. At $x = 4.0$ m it would have a potential energy of 8.0 J and at $x = 2.0$ m it has a potential energy of 4.0 J. Therefore the maximum kinetic energy it can have at $x = 2.0$ m is 4.0 J. This corresponds to a speed of

$$v = \sqrt{\frac{2(4.0\text{ J})}{0.20\text{ kg}}} = 6.3\text{ m/s}$$

Assess: This is a reasonable speed.

Section 10.6 Force and Potential Energy

10.30. Model: Use the definition $F_s = -dU/ds$.

Visualize: Please refer to Figure EX10.30.

Solve: F_x is the negative of the slope of the potential energy graph at position x. Between $x = 0$ cm and $x = 10$ cm the slope is

$$\text{slope} = (U_\text{f} - U_\text{i})/(x_\text{f} - x_\text{i}) = (0\text{ J} - 10\text{ J})/(0.10\text{ m} - 0.0\text{ m}) = -100\text{ N}$$

Thus, $F_x = 100$ N at $x = 5$ cm. The slope between $x = 10$ cm and $x = 20$ cm is zero, so $F_x = 0$ N at $x = 15$ cm. Between 20 cm and 40 cm,

$$\text{slope} = (10\text{ J} - 0\text{ J})/(0.40\text{ m} - 0.20\text{ m}) = 50\text{ N}$$

At $x = 25$ cm and $x = 35$ cm, therefore, $F_x = -50$ N.

10.31. Model: Use the definition $F_s = -dU/ds$.

Visualize: Please refer to Figure EX10.31.

Solve: F_x is the negative of the slope of the potential energy graph at position x.

$$F_x = -\left(\frac{dU}{dx}\right)$$

Between $y = 0$ m and $x = 1$ m, the slope is

$$\text{slope} = (U_\text{f} - U_\text{i})/(x_\text{f} - x_\text{i}) = (60\text{ J} - 0\text{ J})/(1\text{ m} - 0\text{ m}) = 60\text{ N}$$

Thus, $F_x = -60$ N at $x = 1$ m. Between $x = 1$ m and $x = 5$ m, the slope is

$$\text{slope} = (U_\text{f} - U_\text{i})/(x_\text{f} - x_\text{i}) = (0\text{ J} - 60\text{ J})/(5\text{ m} - 1\text{ m}) = -15\text{ N}$$

Thus, $F_x = 15$ N at $x = 4$ m.

10.32. Model: Use the negative derivative of the potential energy to determine the force acting on a particle.
Solve: The y-component of the force is

$$F_y = -\frac{dU}{dy} = -\frac{d}{dy}(4y^3) = -12y^2$$

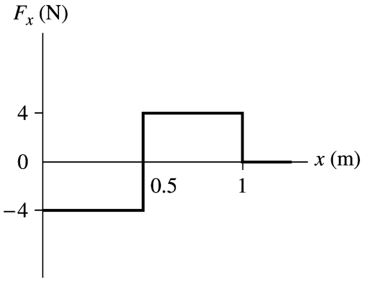
At $y = 0$ m, $F_y = 0$ N; at $y = 1$ m, $F_y = -12$ N; and at $y = 2$ m, $F_y = -48$ N.

10.33. Model: Use the negative derivative of the potential energy to determine the force acting on a particle.
Solve: **(b)** The x-component of the force is

$$F_x = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{10}{x}\right) = \frac{10}{x^2}$$

$$F(x = 2\text{ m}) = \left.\frac{10}{x^2}\right|_{x=2\text{ m}} = 2.5\text{ N}, F(x = 5\text{ m}) = \left.\frac{10}{x^2}\right|_{x=5\text{ m}} = 0.40\text{ N}, F(= 8\text{ m}) = \left.\frac{10}{x^2}\right|_{x=8\text{ m}} = 0.16\text{ N}$$

10.34. Model: Use the relationship between a conservative force and potential energy.
Visualize: We will obtain F_x as a function of x by using the calculus technique of differentiation.
Solve: For the interval $0\text{ m} < x < 0.5\text{ m}$, $U = +4x$, and for the interval $0.5\text{ m} < x < 1.0\text{ m}$, $U = -4x + 4$, where x is in meters. The derivatives give $F_x = -4$ N and $F_x = +4$ N, respectively. The slope is zero for $x \geq 1$ m.



Section 10.7 Conservative and Nonconservative Forces

10.35. Model: Use the definition of work for a constant force $\vec{F}$, $W = \vec{F} \cdot \Delta\vec{s}$, where $\Delta\vec{s}$ is the displacement.

Visualize: Please refer to Figure EX10.35. The force $\vec{F} = (6\hat{i} + 8\hat{j})$ N on the particle is constant.

Solve: (a) $W_{\text{ABD}} = W_{\text{AB}} + W_{\text{BD}} = \vec{F} \cdot (\Delta\vec{s})_{\text{AB}} + \vec{F} \cdot (\Delta\vec{s})_{\text{BD}}$

$$= (6\hat{i} + 8\hat{j})\text{ N} \cdot (3\hat{i})\text{ m} + (6\hat{i} + 8\hat{j})\text{ N} \cdot (4\hat{j})\text{ m} = 18\text{ J} + 32\text{ J} = 50\text{ J}$$

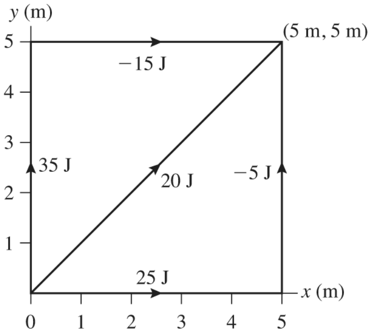
(b) $W_{\text{ACD}} = W_{\text{AC}} + W_{\text{CD}} = \vec{F} \cdot (\Delta\vec{s})_{\text{AC}} + \vec{F} \cdot (\Delta\vec{s})_{\text{CD}}$

$$= (6\hat{i} + 8\hat{j})\text{ N} \cdot (4\hat{j})\text{ m} + (6\hat{i} + 8\hat{j})\text{ N} \cdot (3\hat{j})\text{ m} = 32\text{ J} + 18\text{ J} = 50\text{ J}$$

(c) $W_{\text{AD}} = \vec{F} \cdot (\Delta\vec{s})_{\text{AD}} = (6\hat{i} + 8\hat{j})\text{ N} \cdot (3\hat{i} + 4\hat{j})\text{ m} = 18\text{ J} + 32\text{ J} = 50\text{ J}$

The force is conservative because the work done is independent of the path.

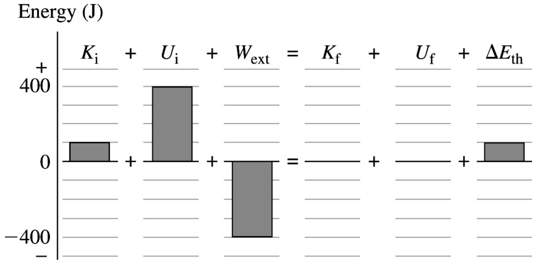
10.36. **Model:** The particle has no thermal energy.
Visualize:



Solve: **(a)** We calculate the work done from (0 m, 0 m) to (5 m, 5 m) along three paths and show they are the same, so we conclude the force is conservative. 25 J – 5 J = 30 J – 15 J = 20 J.
(b) Since the work required along either path is 20 J the particle has 20 J more potential energy at the point (5 m, 5 m). Since it was zero at the origin, it is 20 J at (5 m, 5 m).

Section 10.8 The Energy Principle Revisited

10.37. **Visualize:** The system loses 400 J of potential energy. In the process of losing this energy, it does 400 J of work on the environment, which means $W_{\text{ext}} = -400$ J. Since the thermal energy increases 100 J, we have $\Delta E_{\text{th}} = 100$ J, which must have been 100 J of kinetic energy originally. This is shown in the energy bar chart.

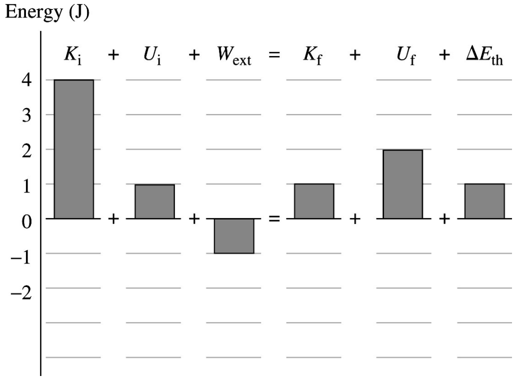


10.38. **Solve:** The sum of the heights of the bars on the left side must equal the sum of the heights of the bars on the right. On the left we have 1 J + 1 J + 4 J = 6 J, so K_f must be 2 J for the total to be 6 J on the right.

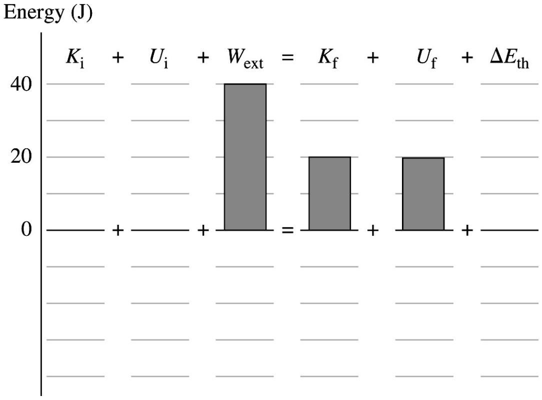
10.39. **Solve:** Please refer to Figure EX10.39. The energy conservation equation yields

$$K_i + U_i + W_{\text{ext}} = K_f + U_f + \Delta E_{\text{th}} \quad \Rightarrow \quad 4\text{ J} + 1\text{ J} + W_{\text{ext}} = 1\text{ J} + 2\text{ J} + 1\text{ J} \quad \Rightarrow \quad \text{W}_{\text{ext}} = -1\text{ J}$$

Thus, the work done to the environment is -1 J. In other words, 1 J of energy is transferred from the system into the environment. This is shown in the energy bar chart.



10.40. Visualize: The tension of 20.0 N in the cable is an external force that does work on the block $W_{\text{ext}} = (20.0\text{ N})(2.00\text{ m}) = 40.0\text{ J}$, increasing the gravitational potential energy of the block. We placed the origin of our coordinate system on the initial resting position of the block, so we have $U_{\text{i}} = 0\text{ J}$ and $U_{\text{f}} = mgy_{\text{f}} = (1.02\text{ kg})(9.8\text{ m/s}^2)(2.00\text{ m}) = 20.0\text{ J}$. Also, $K_{\text{i}} = 0\text{ J}$, and $\Delta E_{\text{th}} = 0\text{ J}$. The energy bar chart shows the energy transfers and transformations.



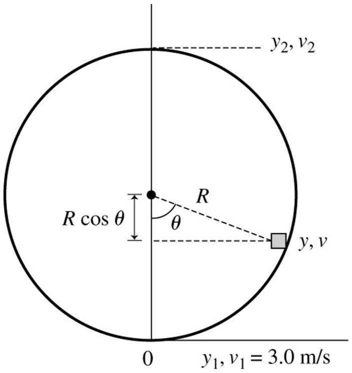
Solve: The conservation of energy equation is

$$K_{\text{i}} + U_{\text{i}} + W_{\text{ext}} = K_{\text{f}} + U_{\text{f}} + \Delta E_{\text{th}} \quad \Rightarrow \quad 0\text{ J} + 0\text{ J} + 40.0\text{ J} = \tfrac{1}{2}mv_{\text{f}}^2 + 20.0\text{ J} + 0\text{ J}$$

$$v_{\text{f}} = \sqrt{(20.0\text{ J})(2)/(1.02\text{ kg})} = 6.26\text{ m/s}$$

Problems

10.41. Model: For the ice cube sliding around the inside of a smooth pipe, the sum of the kinetic and gravitational potential energy does not change. There is no friction.
Visualize:



We use a coordinate system with the origin at the bottom of the pipe, that is, $y_1 = 0$. The radius R of the pipe is 10 cm, and therefore $y_{\text{top}} = y_2 = 2R = 0.20\text{ m}$. At an arbitrary angle θ, measured counterclockwise from the bottom of the circle, $y = R - R\cos\theta$.

Solve: The energy conservation equation $K_2 + U_{\text{g}2} = K_1 + U_{\text{g}1}$ is

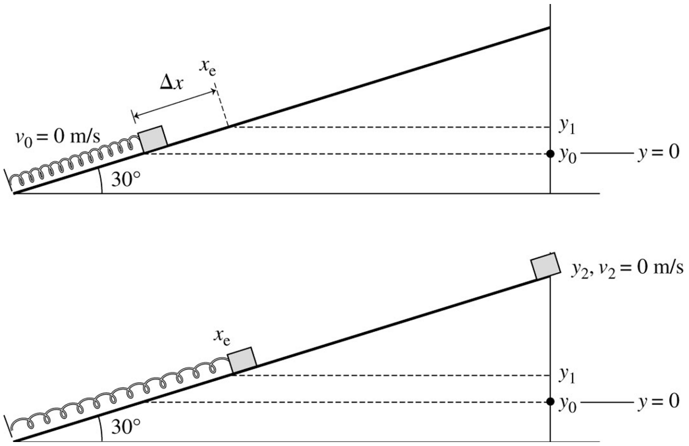
$$\Rightarrow \frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + mgy_1$$

$$\Rightarrow v_2 = \sqrt{v_1^2 + 2g(y_1 - y_2)} = \sqrt{(3.0\text{ m/s})^2 + 2(9.8\text{ m/s}^2)(0\text{ m} - 0.20\text{ m})} = 2.25\text{ m/s} \approx 2.3\text{ m/s}$$

Assess: Beginning with a speed of 3.0 m/s at the bottom, the ice cube's potential energy increases and kinetic energy decreases as it gets toward the top of the circle. At the top, its speed is 2.25 m/s. This is reasonable since some of the kinetic energy has been transformed into the ice cube's potential energy.

10.42. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, and therefore the mechanical energy $K + U_s + U_g$ is conserved.

Visualize:



The figure shows a before-and-after pictorial representation. We have chosen to place the origin of the coordinate system at the position where the ice cube has compressed the spring 10 cm. That is, $y_0 = 0$.

Solve: The energy conservation equation $K_2 + U_{s2} + U_{g2} = K_0 + U_{s0} + U_{g0}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(x_e - x_e)^2 + mgy_2 = \frac{1}{2}mv_0^2 + \frac{1}{2}k(x - x_e)^2 + mgy_0$$

Using $v_2 = 0$ m/s, $y_0 = 0$ m, and $v_0 = 0$ m/s,

$$mgy_2 = \frac{1}{2}k(x - x_e)^2 \Rightarrow y_2 = \frac{k(\Delta x)^2}{2\,mg} = h$$

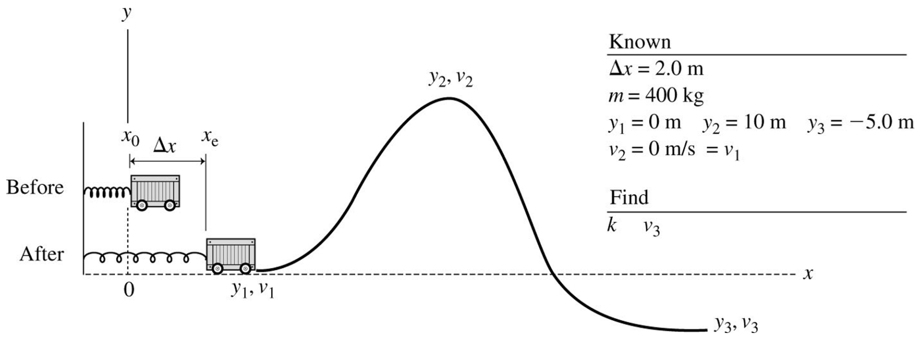
Insert the values given

$$h = \frac{k(\Delta x)^2}{2\,mg} = \frac{(25\text{ N/m})(0.10\text{ m})^2}{2(0.050\text{ kg})(9.8\text{ m/s}^2)} = 25.5\text{ cm} \approx 26\text{ cm}$$

Assess: The net effect of the launch is to transform the potential energy stored in the spring into gravitational potential energy. The block has kinetic energy as it comes off the spring, but we did not need to know this energy to solve the problem. The answer is independent of the angle of the slope.

10.43. Model: Assume an ideal spring that obeys Hooke's law. Also assume zero rolling friction between the roller coaster and the track, and a particle model for the roller coaster. Since no friction is involved, the mechanical energy $K + U_s + U_g$ is conserved.

Visualize:



We have chosen to place the origin of the coordinate system on the end of the spring that is compressed and touches the roller coaster car.

Solve: **(a)** The energy conservation equation for the car going to the top of the hill is

$$K_2 + U_{g2} + U_{s2} = K_0 + U_{g0} + U_{s0} \quad \frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}k(x_e - x_e)^2 = \frac{1}{2}mv_0^2 + mgy_0 + \frac{1}{2}k(x_0 - x_e)^2$$

Noting that $y_0 = 0$ m, $v_2 = 0$ m/s, $v_1 = 0$ m/s, and $|x_0 - x_e| = 2.0$ m, we obtain

$$0 \text{ J} + mgy_2 + 0 \text{ J} = 0 \text{ J} + 0 \text{ J} + \frac{1}{2}k(2.0 \text{ m})^2$$

$$\Rightarrow k = \frac{2mgy_2}{(2.0 \text{ m})^2} = \frac{2(400 \text{ kg})(9.8 \text{ m/s}^2)(10 \text{ m})}{(2.0 \text{ m})^2} = 1.96 \times 10^4 \text{ N/m}$$

We now increase this value for *k* by 10% for safety, giving a value of 2.156×10^4 N/m $\approx 2.2 \times 10^4$ N/m.

(b) The energy conservation equation $K_3 + U_{g3} + U_{s3} = K_0 + U_{g0} + U_{s0}$ is

$$\frac{1}{2}mv_3^2 + mgy_3 + \frac{1}{2}k(x_e - x_e)^2 = \frac{1}{2}mv_0^2 + mgy_0 + \frac{1}{2}k(x_0 - x_e)^2$$

Using $y_3 = -5.0$ m, $v_0 = 0$ m/s, $y_0 = 0$ m, and $|x_0 - x_e| = 2.0$ m, we get

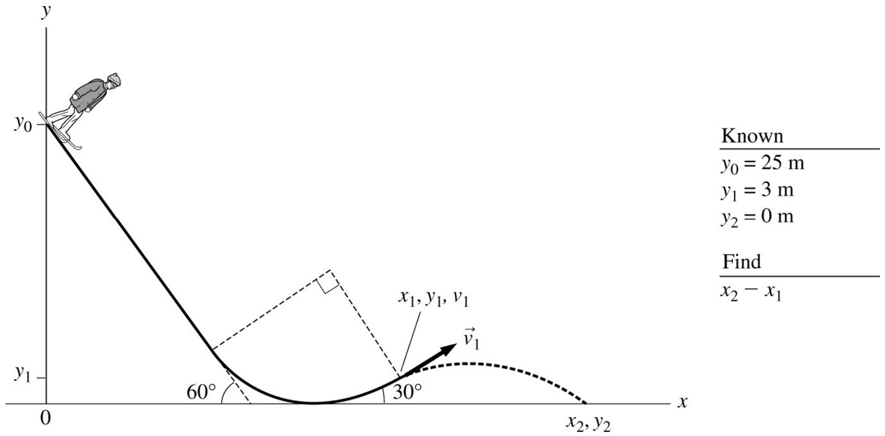
$$\frac{1}{2}mv_3^2 + mg(-5.0 \text{ m}) + 0 \text{ J} = 0 \text{ J} + 0 \text{ J} + \frac{1}{2}k(x_0 - x_e)^2$$

$$\frac{1}{2}(400 \text{ kg})v_3^2 - (400 \text{ kg})(9.8 \text{ m/s}^2)(5.0 \text{ m}) = \frac{1}{2}(2.156 \times 10^4 \text{ N/m})(2.0 \text{ m})^2$$

$$\Rightarrow v_3 = 17.7 \text{ m/s} \approx 18 \text{ m/s}$$

10.44. **Model:** Since there is no friction, the sum of the kinetic and gravitational potential energy does not change. Model Julie as a particle.

Visualize:



We place the coordinate system at the bottom of the ramp directly below Julie's starting position. From geometry, Julie launches off the end of the ramp at a 30º angle.

Solve: Energy conservation: $K_1 + U_{g1} = K_0 + U_{g0} \Rightarrow \frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0$

Using $v_0 = 0$ m/s, $y_0 = 25$ m, and $y_1 = 3$ m, the above equation simplifies to

$$\frac{1}{2}mv_1^2 + mgy_1 = mgy_0 \Rightarrow v_1 = \sqrt{2g(y_0 - y_1)} = \sqrt{2(9.8\text{ m/s}^2)(25\text{ m} - 3\text{ m})} = 20.77\text{ m/s}$$

We can now use kinematic equations to find the touchdown point from the base of the ramp. First we'll consider the vertical motion:

$$y_2 = y_1 + v_{1y}(t_2 - t_1) + \frac{1}{2}a_y(t_2 - t_1)^2 \quad 0\text{ m} = 3\text{ m} + (v_1\sin 30°)(t_2 - t_1) + \frac{1}{2}(-9.8\text{ m/s}^2)(t_2 - t_1)^2$$

$$\Rightarrow (t_2 - t_1)^2 - \frac{(20.77\text{ m/s})\sin 30°}{(4.9\text{ m/s}^2)}(t_2 - t_1) - \frac{(3\text{ m})}{(4.9\text{ m/s}^2)} = 0$$

$$(t_2 - t_1)^2 - (2.119\text{ s})(t_2 - t_1) - (0.6122\text{ s}^2) = 0 \Rightarrow (t_2 - t_1) = 2.377\text{ s}$$

For the horizontal motion:

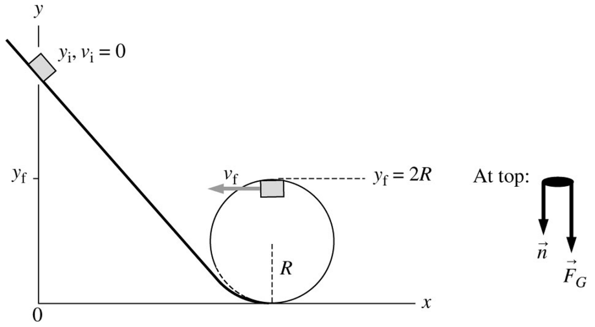
$$x_2 = x_1 + v_{1x}(t_2 - t_1) + \frac{1}{2}a_x(t_2 - t_1)^2$$

$$x_2 - x_1 = (v_1\cos 30°)(t_2 - t_1) + 0\text{ m} = (20.77\text{ m/s})(\cos 30°)(2.377\text{ s}) = 43\text{ m}$$

Assess: Note that we did not have to make use of the information about the circular arc at the bottom that carries Julie through a 90° turn.

10.45. Model: This is a two-part problem. In the first part, we will find the critical velocity for the block to go over the top of the loop without falling off. Since there is no friction, the sum of the kinetic and gravitational potential energy is conserved during the block's motion. We will use this conservation equation in the second part to find the minimum height the block must start from to make it around the loop.

Visualize:



We place the origin of our coordinate system directly below the block's starting position on the frictionless track.

Solve: The free-body diagram on the block implies

$$F_G + n = \frac{mv_c^2}{R}$$

For the block to just stay on track, $n = 0$. Thus the critical velocity v_c is

$$F_G = mg = \frac{mv_c^2}{R} \Rightarrow v_c^2 = gR$$

The block needs kinetic energy $\frac{1}{2}mv_c^2 = \frac{1}{2}mgR$ to go over the top of the loop. We can now use the conservation of mechanical energy equation to find the minimum height h.

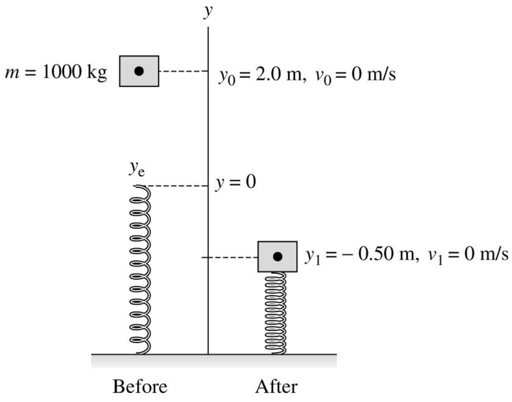
$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Using $v_f = v_c = \sqrt{gR}$, $y_f = 2R$, $v_i = 0$ m/s, and $y_i = h$, we obtain

$$\frac{1}{2}gR + g(2R) = 0 + gh \Rightarrow h = 2.5R$$

10.46. Model: Assume an ideal spring that obeys Hooke's law. Since this is a free-fall problem, the mechanical energy $K + U_g + U_s$ is conserved. Also, model the safe as a particle.

Visualize:



We have chosen to place the origin of our coordinate system at the free end of the spring, which is neither stretched nor compressed. The safe gains kinetic energy as it falls. The energy is then converted into elastic potential energy as the safe compresses the spring. The only two forces are gravity and the spring force, which are both conservative, so energy is conserved throughout the process. This means that the initial energy—as the safe is released—equals the final energy—when the safe is at rest and the spring is fully compressed.

Solve: The conservation of energy equation $K_1 + U_{g1} + U_{s1} = K_0 + U_{g0} + U_{s0}$ is

$$\frac{1}{2}mv_1^2 + mg(y_1 - y_e) + \frac{1}{2}k(y_1 - y_e)^2 = \frac{1}{2}mv_0^2 + mg(y_0 - y_e) + \frac{1}{2}k(y_e - y_e)^2$$

Using $v_1 = v_0 = 0$ m/s and $y_e = 0$ m, the above equation simplifies to

$$mgy_1 + \frac{1}{2}ky_1^2 = mgy_0$$

$$\Rightarrow k = \frac{2mg(y_0 - y_1)}{y_1^2} = \frac{2(1000\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m} - (-0.50\text{ m}))}{(-0.50\text{ m})^2} = 1.96\times10^5\text{ N/m} \approx 2.0\times10^5\text{ N/m}$$

Assess: By equating energy at these two points, we do not need to find how fast the safe was moving when it hit the spring

10.47. Model: Assume the spring is ideal and obeys Hooke's law. Then the potential energy of a stretched spring is $U_{sp} = \frac{1}{2}k(\Delta s)^2$.

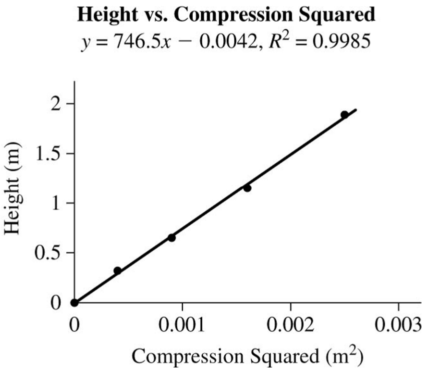
Visualize: The kinetic energy at the beginning is zero, and it is also zero at maximum height, so the spring potential energy at the beginning equals the gravitational potential energy at maximum height. We are given $k = 950$ N/m.

Solve:

$$(U_s)_i = (U_g)_f \Rightarrow \frac{1}{2}k(\Delta s)^2 = mgy$$

Putting this in $y = mx + b$ form $y = \frac{k}{2mg}(\Delta s)^2$ leads us to believe that a graph of y vs. $(\Delta s)^2$ would produce a straight line whose slope is $k/2mg$ and whose intercept is zero.

Compression (m)	Height (m)	Compression Squared (m^2)
0	0	0
0.02	32	0.0004
0.03	65	0.0009
0.04	115	0.0016
0.05	189	0.0025

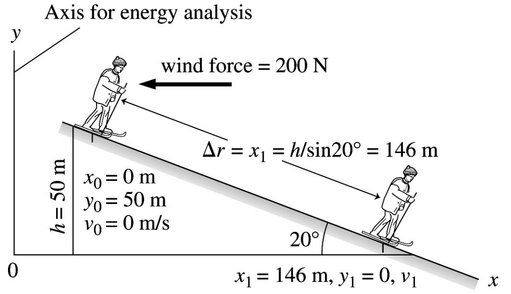


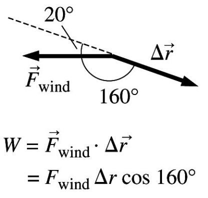
The spreadsheet tells us the fit is very good and that the slope is 746.5 m^{-1}.

$$\frac{k}{2mg} = 746.5\text{ m}^{-1} \Rightarrow m = \frac{k}{2g(746.5\text{ m}^{-1})} = \frac{950\text{ N/m}}{2(9.8\text{ m/s}^2)(746.5\text{ m}^{-1})} = 0.065\text{ kg} = 65\text{ g}$$

Assess: 65 g seems reasonable, and we were happy to get a very small intercept on our best-fit line.

10.48. Model: Model Sam strapped with skis as a particle, and apply the law of conservation of energy.
Visualize:





Solve: **(a)** The conservation of energy equation is

$$K_1 + U_{g1} + \Delta E_{th} = K_0 + U_{g0} + W_{ext}$$

The snow is frictionless, so $\Delta E_{th} = 0$ J. However, the wind is an external force doing work on Sam as he moves down the hill. Thus,

$$W_{ext} = W_{wind} = (K_1 + U_{g1}) - (K_0 + U_{g0})$$
$$= \left(\tfrac{1}{2}mv_1^2 + mgy_1\right) - \left(\tfrac{1}{2}mv_0^2 + mgy_0\right) = \left(\tfrac{1}{2}mv_1^2 + 0\text{ J}\right) - (0\text{ J} + mgy_0) = \tfrac{1}{2}mv_1^2 - mgy_0$$
$$v_1 = \sqrt{2gy_0 + \frac{2W_{wind}}{m}}$$

We compute the work done by the wind as follows:

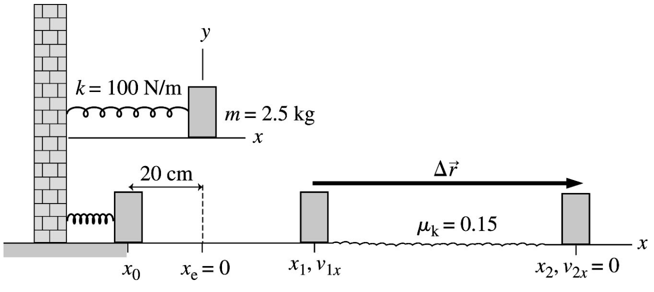
$$W_{wind} = \vec{F}_{wind} \cdot \Delta\vec{r} = F_{wind}\Delta r\cos(160°) = (200\text{ N})(146\text{ m})\cos(160°) = -27{,}400\text{ J}$$

where we have used $\Delta r = h/\sin(20°) = 146$ m. Now we can compute

$$v_1 = \sqrt{2(9.8\text{ m/s}^2)(50\text{ m}) + \frac{2(-27{,}400\text{ J})}{75\text{ kg}}} = 16\text{ m/s}$$

Assess: We used a vertical y-axis for energy analysis, rather than a tilted coordinate system, because U_g is determined by its vertical position.

10.49. Model: Assume an ideal spring that obeys Hooke's law. Model the box as a particle and use the model of kinetic friction.
Visualize:



Solve: When the horizontal surface is frictionless, conservation of energy means

$$\tfrac{1}{2}k(x_0 - x_e)^2 = \tfrac{1}{2}mv_{1x}^2 = K_1 \quad\Rightarrow\quad K_1 = \tfrac{1}{2}(100\text{ N/m})(0.20\text{ m} - 0\text{ m})^2 = 2.0\text{ J}$$

That is, the box is launched with 2.0 J of kinetic energy. It will lose 2.0 J of kinetic energy on the rough surface. The net force on the box is $\vec{F}_{\text{net}} = -\vec{f}_{\text{k}} = -\mu_{\text{k}} mg\hat{i}$. The work–kinetic energy theorem is

$$W_{\text{net}} = \vec{F}_{\text{net}} \cdot \Delta\vec{r} = K_2 - K_1 = 0\text{ J } - 2.0\text{ J} = -2.0\text{ J}$$

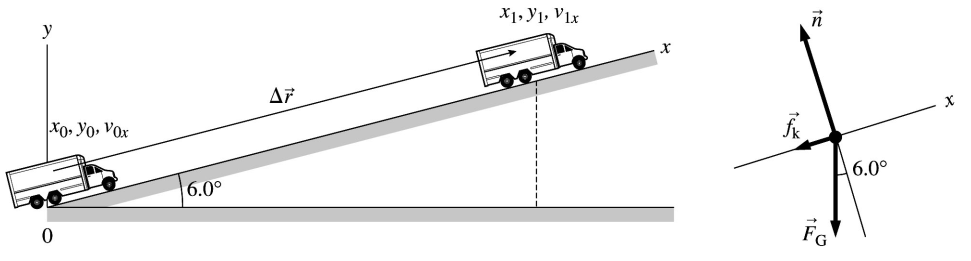
$$(-\mu_{\text{k}} mg)(x_2 - x_1) = -2.0\text{ J}$$

$$(x_2 - x_1) = \frac{2.0\text{ J}}{\mu_{\text{k}} mg} = \frac{2.0\text{ J}}{(0.15)(2.5\text{ kg})(9.8\text{ m/s}^2)} = 0.54\text{ m}$$

Assess: Because the force of friction transforms kinetic energy into thermal energy, energy is transferred out of the box into the environment. In response, the box slows down and comes to rest.

10.50. Model: Identify the truck and the loose gravel as the system. We need the gravel inside the system because friction increases the temperature of the truck and the gravel. We will also use the model of kinetic friction and the conservation of energy equation.

Visualize:



Known
$m = 15{,}000$ kg
$\mu_{\text{k}} = 0.40$
$v_{0x} = 35$ m/s
$v_{1x} = 0$ m/s
$x_0 = 0$ m
$y_0 = 0$ m
$y_1 = x_1 \sin(6.0°)$

Find
x_1

We place the origin of our coordinate system at the base of the ramp in such a way that the x-axis is along the ramp and the y-axis is vertical so that we can calculate potential energy. The free-body diagram of forces on the truck is shown.

Solve: The conservation of energy equation is $K_1 + U_{\text{g}1} + \Delta E_{\text{th}} = K_0 + U_{\text{g}0} + W_{\text{ext}}$. In the present case, $W_{\text{ext}} = 0$ J, $v_{1x} = 0$ m/s, $U_{\text{g}0} = 0$ J, $v_{0x} = 35$ m/s. The thermal energy created by friction is

$$\Delta E_{\text{th}} = f_{\text{k}}(x_1 - x_0) = (\mu_{\text{k}} n)(x_1 - x_0) = \mu_{\text{k}} mg\cos(6.0°)(x_1 - x_0)$$

$$= (0.40)(15{,}000\text{ kg})(9.8\text{ m/s}^2)\cos(6.0°)(x_1 - x_0) = (58{,}478\text{ J/m})(x_1 - x_0)$$

Thus, the energy conservation equation simplifies to

$$0\text{ J} + mgy_1 + (58{,}478\text{ J/m})(x_1 - x_0) = \tfrac{1}{2}mv_{0x}^2 + 0\text{ J} + 0\text{ J}$$

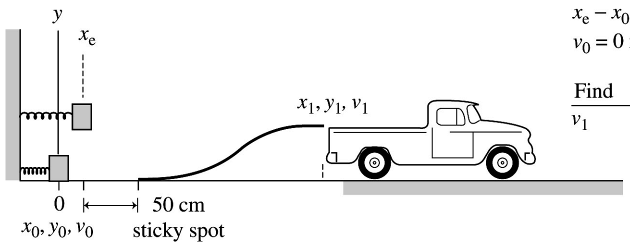
$$(15{,}000\text{ kg})(9.8\text{ m/s}^2)(x_1 - x_0)\sin(6.0°) + (58{,}478\text{ J/m})(x_1 - x_0) = \tfrac{1}{2}(15{,}000\text{ kg})(35\text{ m/s})^2$$

$$(x_1 - x_0) = 124\text{ m} = 0.12\text{ km}$$

Assess: A length of 124 m at a slope of 6° seems reasonable.

10.51. Model: We will use the spring, the package, and the ramp as the system. We will model the package as a particle.

Visualize:



Known

$x_0 = y_0 = 0$ m $m = 2.0$ kg

$x_e - x_0 = 30$ cm $y_1 = 1.0$ m

$v_0 = 0$ m/s $k = 500$ N/m

Find

v_1

We place the origin of our coordinate system on the end of the spring when it is compressed and is in contact with the package to be shot.

Model: (a) The energy conservation equation is

$$K_1 + U_{g1} + U_{s1} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(x_e - x_e)^2 + \Delta E_{th} = \tfrac{1}{2}mv_0^2 + mgy_0 + \tfrac{1}{2}k(\Delta x)^2 + W_{ext}$$

Using $y_1 = 1.0$ m, $\Delta E_{th} = 0$ J (the frictionless ramp), $v_0 = 0$ m/s, $y_0 = 0$ m, $\Delta x = 30$ cm, and $W_{ext} = 0$ J, we get

$$\tfrac{1}{2}mv_1^2 + mg(1.0\text{ m}) + 0\text{ J} + 0\text{ J} = 0\text{ J} + 0\text{ J} + \tfrac{1}{2}k(0.30\text{ m})^2 + 0\text{ J}$$

$$\tfrac{1}{2}(2.0\text{ kg})v_1^2 + (2.0\text{ kg})(9.8\text{ m/s}^2)(1.0\text{ m}) = \tfrac{1}{2}(500\text{ N/m})(0.30\text{ m})^2$$

$$v_1 = 1.7\text{ m/s}$$

(b) How high can the package go after crossing the sticky spot? If the package can reach $y_1 \geq 1.0$ m before stopping $(v_1 = 0)$, then it makes it. But if $y_1 < 1.0$ m when $v_1 = 0$, the package does not make it. The friction of the sticky spot generates thermal energy

$$\Delta E_{th} = (\mu_k mg)\Delta x = (0.30)(2.0\text{ kg})(9.8\text{ m/s}^2)(0.50\text{ m}) = 2.94\text{ J}$$

The energy conservation equation is now

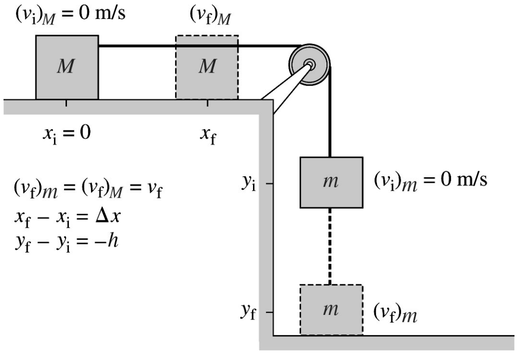
$$\tfrac{1}{2}mv_1^2 + mgy_1 + \Delta E_{th} = \tfrac{1}{2}k(\Delta x)^2$$

If we set $v_1 = 0$ m/s to find the highest point the package can reach, we get

$$y_1 = \left(\tfrac{1}{2}k\Delta x^2 - \Delta E_{th}\right)\Big/(mg) = [\tfrac{1}{2}(500\text{ N/m})(0.30\text{ m})^2 - 2.94\text{ J}]/[(2.0\text{ kg})(9.8\text{ m/s}^2)] = 0.998\text{ m}$$

The package does not make it. It just barely misses.

10.52. Model: Model the two blocks as particles. The two blocks make our system.
Visualize:



We place the origin of our coordinate system at the location of the 3.0 kg block.

Solve: (a) The conservation of energy equation gives $K_f + U_{gf} + \Delta E_{th} = K_i + U_{gi} + W_{ext}$. The thermal energy is $\Delta E_{th} = \mu_k Mg\Delta x$ and the external work done is $W_{ext} = 0$. The initial potential energy is $U_{gi} = my_i$, and the final potential energy is $U_{gf} = my_f$, where we have ignored the gravitational potential energy of block M because its height does not change. The initial and final kinetic energy are $K_i = 0$, and $K_f = \frac{1}{2}(M+m)v_f^2$, respectively. The energy conservation equation thus takes the form

$$\tfrac{1}{2}(m_3 + m_2)v_f^2 + m_3 y_{table} + m_2 g y_f + \mu_k m_3 g \Delta x = m_3 y_{table} + m_2 g y_i$$

Note that $\Delta x = -\Delta y = h$ because the blocks are constrained by the cable to move the same distance. Solving for v_f gives

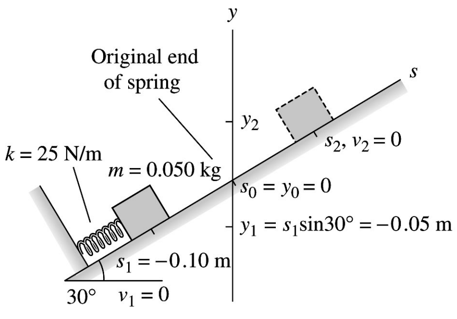
$$v_f = \sqrt{\frac{2g}{M+m}(-m\Delta y - \mu_k M \Delta x)} = \sqrt{\frac{2gh}{M+m}(m - \mu_k M)}$$

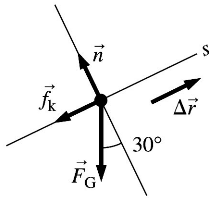
(b) If the table is frictionless, this expression takes the form

$$v_f = \sqrt{\frac{2gmh}{M+m}}$$

Assess: It is reasonable that the speed is reduced when friction is present compared with when there is no friction.

10.53. Model: Model the ice cube as a particle, the spring as an ideal that obeys Hooke's law, and the law of conservation of energy.
Visualize:





Solve: (a) The normal force does no work and the slope is frictionless, so mechanical energy is conserved. We've drawn two separate axes: a vertical y-axis to measure potential energy and a tilted s-axis to measure distance along the slope. Both have the same origin which is at the point where the spring is not compressed. Thus, the two axes are related by $y = s\sin\theta$. Also, this choice of origin makes the elastic potential energy simply $U_s = \frac{1}{2}k(s-s_0)^2 = \frac{1}{2}ks^2$. Because energy is conserved, we can relate the initial point—with the spring compressed—to the final point where the ice cube is at maximum height. We do *not* need to find the speed with which it leaves the spring. We have

$$K_2 + U_{g2} + U_{s2} = K_1 + U_{g1} + U_{s1}$$

$$\tfrac{1}{2}mv_2^2 + mgy_2 + \tfrac{1}{2}ks_0^2 = \tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}ks_1^2$$

It is important to note that at the final point, when the ice cube is at y_2, the end of the spring is only at s_0. The spring does *not* stretch to s_2, so U_{s2} is *not* $\frac{1}{2}ks_2^2$. Three of the terms are zero, leaving

$$mgy_2 = +mgy_1 + \tfrac{1}{2}ks_1^2 \quad \Rightarrow \quad y_2 - y_1 = \Delta y = \text{height gained} = \frac{ks_1^2}{2mg} = 0.255\ \text{m} = 25.5\ \text{cm}$$

The distance traveled is $\Delta s = \Delta y/\sin(30°) = 0.51$ m.

(b) Using the energy equation and the expression for thermal energy:

$$K_2 + U_{g2} + U_{s2} + \Delta E_{\text{th}} = K_1 + U_{g1} + U_{s1} + W_{\text{ext}},\ \Delta E_{\text{th}} = f_k\Delta s = \mu_k n\Delta s$$

From the free-body diagram,

$$(\vec{F}_{\text{net}})_y = 0\ \text{N} = n - mg\cos(30°) \quad \Rightarrow \quad n = mg\cos(30°)$$

Now, having found $\Delta E_{\text{th}} = \mu_k mg\cos(30°)\Delta s$, the energy equation can be written

$$0\ \text{J} + mgy_2 + 0\ \text{J} + \mu_k mg\cos(30°)\Delta s = 0\ \text{J} + mgy_1 + \tfrac{1}{2}ks_1^2 + 0\ \text{J}$$

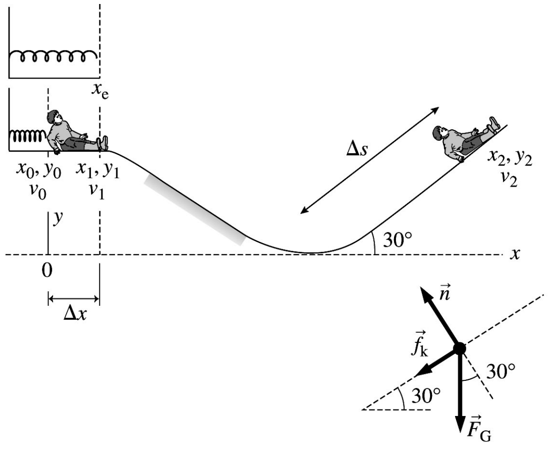
$$mg(y_2 - y_1) - \tfrac{1}{2}ks_1^2 + \mu_k mg\cos(30°)\Delta s = 0$$

Using $\Delta y = \Delta s\sin(30°)$, the above equation simplifies to

$$mg\Delta s\sin(30°) + \mu_k mg\cos(30°)\Delta s = \tfrac{1}{2}ks_1^2 \quad \Rightarrow \quad \Delta s = \frac{ks_1^2}{2mg[\sin(30°) + \mu_k\cos(30°)]} = 0.38\ \text{m}$$

10.54. Model: Assume an ideal spring, so Hooke's law is obeyed. Treat the physics student as a particle and apply the law of conservation of energy. Our system comprises the spring, the student, and the ground. We also use the model of kinetic friction.

Visualize: We place the origin of the coordinate system on the ground directly below the end of the compressed spring that is in contact with the student.



Known

$x_0 = 0$ m $\quad v_0 = 0$ m/s

$x_1 - x_0 = 0.50$ m $\quad k = 80{,}000$ N/m

$y_0 = y_1 = 10$ m $\quad m = 100$ kg

$\mu_k = 0.15 \quad v_2 = 0$ m/s

Find

$v_1 \quad \Delta s = y_2/\sin 30°$

Solve: **(a)** The energy conservation equation gives

$$K_1 + U_{g1} + U_{s1} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\tfrac{1}{2}mv_1^2 + mgy_1 + \tfrac{1}{2}k(x_1 - x_e)^2 + 0\text{ J} = \tfrac{1}{2}mv_0^2 + mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2 + 0\text{ J}$$

Since $y_1 = y_0 = 10$ m, $x_1 = x_e$, $v_0 = 0$ m/s, $k = 80{,}000$ N/m, $m = 100$ kg, and $(x_1 - x_0) = 0.5$ m,

$$\tfrac{1}{2}mv_1^2 = \tfrac{1}{2}k(x_1 - x_0)^2 \quad\Rightarrow\quad v_1 = \sqrt{\frac{k}{m}}(x_1 - x_0) = \sqrt{\frac{80{,}000\text{ N}/m}{100\text{ kg}}}(0.50\text{ m}) = 14\text{ m/s}$$

(b) Friction creates thermal energy. Applying the conservation of energy equation once again:

$$K_2 + U_{g2} + U_{s2} + \Delta E_{th} = K_0 + U_{g0} + U_{s0} + W_{ext}$$

$$\tfrac{1}{2}mv_2^2 + mgy_2 + 0\text{ J} + f_k\Delta s = 0\text{ J} + mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2 + 0\text{ J}$$

With $v_2 = 0$ m/s and $y_2 = \Delta s\sin(30°)$, the above equation is simplified to

$$mg\Delta s\sin(30°) + \mu_k n\Delta s = mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2$$

From the free-body diagram for the physics student, we see that $n = F_G\cos(30°) = mg\cos(30°)$. Thus, the conservation of energy equation gives

$$\Delta s[mg\sin(30°) + \mu_k mg\cos(30°)] = mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2$$

Using $m = 100$ kg, $k = 80{,}000$ N/m, $(x_1 - x_0) = 0.50$ m, $y_0 = 10$ m, and $\mu_k = 0.15$, we get

$$\Delta s = \frac{mgy_0 + \tfrac{1}{2}k(x_1 - x_0)^2}{mg[\sin(30°) + \mu_k\cos(30°)]} = 32\text{ m}$$

Assess: $y_2 = \Delta s\sin(30°) = 16$ m, which is greater than $y_0 = 10$ m. The higher value is due to the transformation of the spring energy into gravitational potential energy.

10.55. Model: The potential energy of two nucleons interacting via the strong force is

$$U = U_0[1 - e^{-x/x_0}]$$

where x is the distance between the centers of the two nucleons, $x_0 = 2.0\times10^{-15}$ m, and $U_0 = 6.0\times10^{-11}$ J.

Visualize: The neutrons are released from rest, so the initial kinetic energy is zero.

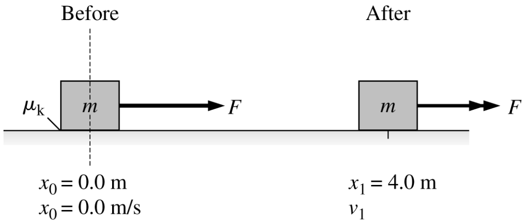
Solve: For $x = 5.0\times10^{-15}$ m, $U = 55.1\times10^{-12}$ J. Due to conservation of total energy, the potential energy when $x = 5.0\times10^{-15}$ J is transformed into kinetic energy until $x = \text{twice the radius} = 1.0\times10^{-15}$ m. At this separation, $U = 23.6\times10^{-11}$ J. Thus,

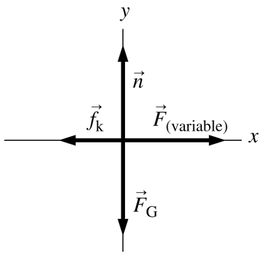
$$K_f + U_f = K_i + U_i$$

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 + 2.36\times10^{-11}\text{ J} = 5.51\times10^{-11}\text{ J} \Rightarrow v = 1.37\times10^8\text{ m/s}$$

Assess: A speed of 1.94×10^8 m/s is approximately 0.65 *c* where *c* is the speed of light. This speed is understandable for the present model.

10.56. Model: Model the block as a particle.
Visualize: The system is the block.





Solve: A preliminary calculation: use Newton's second law in the y direction to see that $n = mg$ so $f_k = \mu_k mg$. Now find the work done on the block by the forces on it.

$$W_{\text{net}} = \int F_{\text{net}}dx = \int_0^4((20-5x)-\mu_k mg)dx = \left[20x-\tfrac{5}{2}x^2-(0.25)(2.6\text{ kg})(9.8\text{ m/s}^2)x\right]_0^4 = 14.52\text{ J}$$

Use the work–kinetic energy theorem: $W_{\text{net}} = \Delta K$ remembering that $K_i = 0$.

$$v = \sqrt{\frac{2W_{\text{net}}}{m}} = \sqrt{\frac{2(14.52\text{ J})}{2.6\text{ kg}}} = 3.3\text{ m/s}$$

Assess: This seems like a reasonable speed for a 2.6 kg block after pulling it 4.0 m.

10.57. Solve: (a) The equilibrium positions are located at points where $\frac{dU}{dx} = 0$.

$$\frac{dU}{dx} = 0 = 1 + 2\cos(2x) \Rightarrow \cos(2x) = -\frac{1}{2}$$
$$\Rightarrow x = \frac{1}{2}\cos^{-1}\left(-\frac{1}{2}\right)$$

Note that $-\frac{1}{2}$ is in radians and x is in meters. The function $\cos^{-1}\left(-\frac{1}{2}\right)$ may have values $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$. Thus there are two values of x,

$$x_1 = \frac{\pi}{3} \text{ and } x_2 = \frac{2\pi}{3}$$

within the interval $0\text{ m} \le x \le \pi$ m.

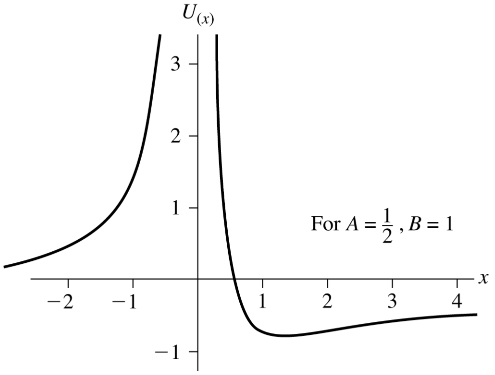
(b) A point of stable equilibrium corresponds to a local minimum, while a point of unstable equilibrium corresponds to a local maximum. Compute the concavity of $U(x)$ at the equilibrium positions to determine their stability.

$$\frac{d^2U}{dx^2} = -4\sin(2x)$$

At $x_1 = \frac{\pi}{3}$, $\frac{d^2U}{dx^2}(x_1) = -4\left(\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$. Since $\frac{d^2U}{dx^2}(x_1) < 0$, $x_1 = \frac{\pi}{3}$ is a local maximum, so $x_1 = \frac{\pi}{3}$ is a point of unstable equilibrium.

At $x_2 = \frac{2\pi}{3}$, $\frac{d^2U}{dx^2}(-x_2) = -4\left(-\frac{\sqrt{3}}{2}\right) = +2\sqrt{3}$. Since $\frac{d^2U}{dx^2} > 0$, $x_2 = \frac{2\pi}{3}$ is a local minimum, so $x_2 = \frac{2\pi}{3}$ is a point of stable equilibrium.

10.58. Visualize: The potential energy is not defined for $x = 0$. The equilibrium points are at the bottom of valleys or the tops of hills of the potential energy graph.



Solve: (a) The equilibrium points are where the slope of the $U(x)$ graph is horizontal, so we take the derivative and set it equal to zero.

$$\frac{dU}{dx} = \frac{-2A}{x^3} + \frac{B}{x^2} = 0 \Rightarrow x = \frac{2A}{B}$$

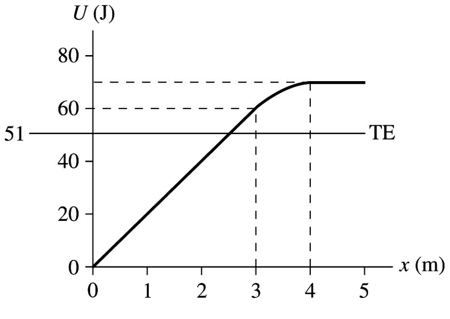
(b) We see that it is a stable equilibrium from looking at the graph, but we check this by taking the second derivative and evaluating it at $x = 2A/B$.

$$\frac{d^2U}{dx^2} = \frac{6A}{x^4} - \frac{2B}{x^3}$$

$$\text{at } x = \frac{2A}{B}, \quad \frac{d^2U}{dx^2} = 6A\left(\frac{B}{2A}\right)^4 - 2B\left(\frac{B}{2A}\right)^3 = \frac{1}{8}\frac{B^4}{A^3}$$

This is positive for positive A and B, so the equilibrium is stable.
Assess: Technically dU/dx is also zero at $x = \infty$, but it is difficult to call that an equilibrium position (it would be neither stable nor unstable since U is not maximized or minimized there.

10.59. Model: The force is conservative, so it has a potential energy.
Visualize: The graph of the potential energy is the negative integral of the conservative force.
Solve: (a) The definition of potential energy is $\Delta U = -W(\text{i} \rightarrow \text{f})$. In addition, work is the area under the force-versus-displacement graph. Thus $\Delta U = U_\text{f} - U_\text{i} = -(\text{area under the force curve})$. Since $U_\text{i} = 0$ at $x = 0$ m, the potential energy at position x is $U(x) = -(\text{area under the force curve from 0 to } x)$. From 0 m to 3 m, the area increases linearly from 0 Nm to -60 Nm, so U increases from 0 J to 60 J. That is, $U(x = 1.0 \text{ m}) = 20$ J, $U(x = 2.0 \text{ m}) = 40$ J, $U(x = 3.0 \text{ m}) = 60$ J. At $x = 4$ m, the area is -70 J. Thus $U = 70$ J at $x = 4$ m, and U doesn't change after that since the force is then zero. Between 3 m and 4 m, where F changes linearly, U must have a quadratic dependence on x (i.e., the potential energy curve is a parabola). This information is shown on the potential energy graph below.

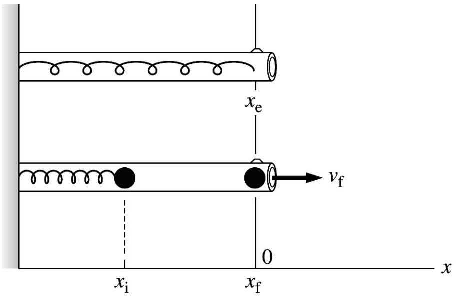


(b) Mechanical energy is $E = K + U$. From the graph, $U = 20$ J at $x = 1.0$ m.

The kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.100\text{ kg})(25\text{ m/s})^2 = 31.25$ J. Thus $TE = 51.25$ J. The turning point occurs where the total energy line crosses the potential energy curve. We can see from the graph that this is at approximately 2.5 m. For a more accurate value, the potential energy function is $U = 20x$ J. The TE line crosses at the point where $20x = 51.25,$ which is $x = 2.6$ m.

10.60. Model: A "sprong" obeys the force law $F_x = -q(x - x_e)^3$, where q is the sprong constant and x_e is the equilibrium position.

Visualize: We place the origin of the coordinate system on the free end of the sprong, that is, $x_e = x_f = 0$ m.



Known

$x_i = -0.10$ m $v_i = 0$ m/s

$x_f = x_e = 0$ m

$m = 20$ g $q = 40{,}000$ N/m^3

Find

v_f

Solve: (a) The units of q are N/m^3.

(b) Since $F_x = -dU/dx,$ we have $U(x) = -\int F_x dx = -\int_0^x (-qx^3)dx = qx^4/4.$

(c) Applying the energy conservation equation to the ball and sprong system gives

$$K_f + U_f = K_i + U_i$$

$$\tfrac{1}{2}mv_f^2 + 0\text{ J} = 0\text{ J} + \tfrac{1}{4}qx_i^4$$

$$v_f = \sqrt{\frac{qx^4}{2m}} = \sqrt{\frac{(40{,}000\text{ N/m}^3)(-0.10\text{ m})^4}{2(0.020\text{ kg})}} = 10\text{ m/s}$$

10.61. Model: Since there is a potential energy then the force is conservative.

Visualize: The force is the negative derivative of the potential energy.

Solve:

$$F_x = -\frac{dU_x}{dx} = -\frac{d\left(Ax^2 + B\sin\left(\frac{\pi}{L}x\right)\right)}{dx} = -2Ax - B\left(\tfrac{\pi}{L}\right)\cos\left(\tfrac{\pi}{L}x\right)$$

(a) Evaluate this expression at $x = 0$: $F_x(0) = -\pi B/L.$

(b) Evaluate the expression at $x = L/2$: $F_x(L/2) = -AL.$

(c) Evaluate the expression at $x = L$: $F_x(L) = -2AL + \pi B/L.$

10.62. Visualize: The potential energy is the negative integral of the conservative force. Use SI units.

Solve:

$$U_x = -\int F_x dx = -\int (3x^2 - 5x)dx = \left(-x^3 + \tfrac{5}{2}x^2 + C\right)\text{ J}$$

where C is the constant of integration and comes in joules, just as the other terms do.

10.63. Solve: (a)

$$W=\left[\int_0^a F_x dx\right]_{y=0}+\left[\int_0^b F_y dy\right]_{x=a}=\left[\int_0^a 2xydx\right]_{y=0}+\left[\int_0^b x^2 dy\right]_{x=a}=0+a^2\big[y\big]_0^b=a^2b$$

(b)

$$W=\left[\int_0^b F_y dy\right]_{x=0}+\left[\int_0^a F_x dx\right]_{y=b}=\left[\int_0^b x^2 dy\right]_{x=0}+\left[\int_0^a 2xydx\right]_{y=b}=0+b\big[x^2\big]_0^a=a^2b$$

(c) We'll conclude the force is conservative because we got the same answer for both paths to an arbitrary point. (In reality the work done must be the same for **all** paths, not just two.)

10.64. Solve: (a)

$$W=\left[\int_0^a F_x dx\right]_{y=0}+\left[\int_0^b F_y dy\right]_{x=a}=\left[\int_0^a 2xydx\right]_{y=0}+\left[\int_0^b 3ydy\right]_{x=a}=0+\left[\tfrac{3}{2}y^2\right]_0^b=\tfrac{3}{2}b^2$$

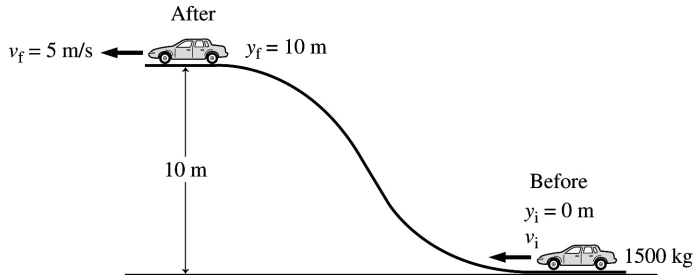
(b)

$$W=\left[\int_0^b F_y dy\right]_{x=0}+\left[\int_0^a F_x dx\right]_{y=b}=\left[\int_0^b 3ydy\right]_{x=0}+\left[\int_0^a 2xydx\right]_{y=b}=\left[\tfrac{3}{2}y^2\right]_0^b+b\left[x^2\right]_0^a=\tfrac{3}{2}b^2+a^2b$$

(c) Because the answers for the two different paths are not the same we are assured that the force is not conservative.

10.65. A 2.5 kg ball is thrown upward at a speed of 4.0 m/s from a height of 82 cm above a vertical spring. When the ball comes down it lands on and compresses the spring. If the spring has a spring constant of $k = 600$ N/m, by how much is it compressed?

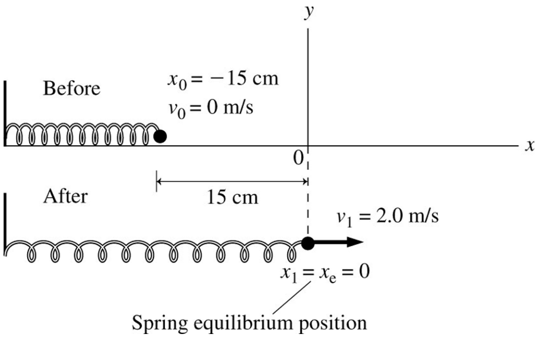
10.66. (a) A 1500 kg auto coasts up a 10.0 m high hill and reaches the top with a speed of 5.0 m/s. What initial speed must the auto have had at the bottom of the hill?
(b)



We place the origin of our coordinate system at the bottom of the hill.
(c) The solution of the equation is $v_i = 14.9 \text{ m/s} \approx 15 \text{ m/s}$. This is approximately 30 mph and is a reasonable value for the speed at the bottom of the hill.

10.67. (a) A spring gun is compressed 15 cm to launch a 200 g ball on a horizontal, frictionless surface. The ball has a speed of 2.0 m/s as it loses contact with the spring. Find the spring constant of the gun.

(b)



We place the origin of our coordinate system on the free end of the spring in the equilibrium position. Because the surface is frictionless, the mechanical energy for the system $(\text{ball} + \text{spring})$ is conserved.

(c) The conservation of energy equation is

$$K_f + U_{sf} = K_i + U_{si}$$

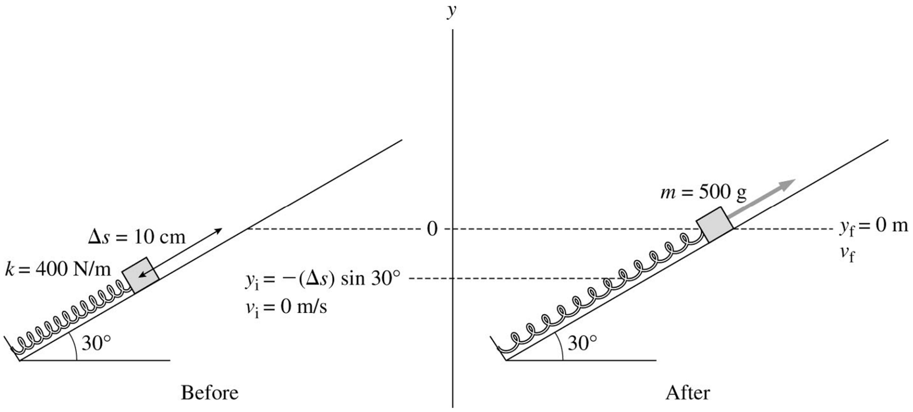
$$\frac{1}{2}mv_1^2 + \frac{1}{2}k(0\text{ m})^2 = \frac{1}{2}m(0\text{ m/s})^2 + \frac{1}{2}k(-0.15\text{ m})^2$$

$$(0.200\text{ kg})(2.0\text{ m/s})^2 = k(-0.15\text{ m})^2$$

$$k = 36\text{ N/m}$$

10.68. (a) A spring with spring constant 400 N/m is anchored at the bottom of a frictionless 30° incline. A 500 g block is pressed against the spring, compressing the spring by 10 cm, then released. What is the speed with which the block is launched up the incline?

(b) The origin is placed at the end of the uncompressed spring. This is the point from which the block is launched as the spring expands.

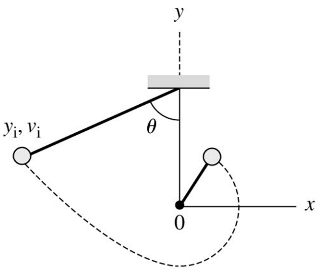


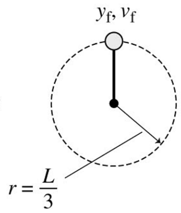
(c) Solving the energy conservation equation, we get $v_f = 2.6$ m/s.

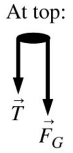
Challenge Problems

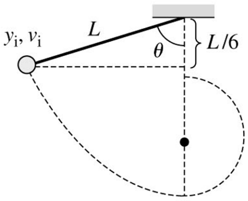
10.69. Model: This is a two-part problem. In the first part, we will find the critical velocity for the ball to go over the top of the peg without the string going slack. Using the energy conservation equation, we will then obtain the gravitational potential energy that gets transformed into the critical kinetic energy of the ball, thus determining the angle θ.

Visualize:









We place the origin of our coordinate system on the peg. This choice will provide a reference to measure gravitational potential energy. For θ to be minimum, the ball will just go over the top of the peg.

Solve: The two forces in the free-body force diagram provide the centripetal acceleration at the top of the circle. Newton's second law at this point is

$$F_G + T = \frac{mv^2}{r}$$

where T is the tension in the string. The critical speed v_c at which the string goes slack is found when $T \to 0$. In this case,

$$mg = \frac{mv_C^2}{r} \Rightarrow v_C^2 = gr = gL/3$$

The ball should have kinetic energy at least equal to

$$\frac{1}{2}mv_C^2 = \frac{1}{2}mg\left(\frac{L}{3}\right)$$

for the ball to go over the top of the peg. We will now use the conservation of mechanical energy equation to get the minimum angle θ. The equation for the conservation of energy is

$$K_f + U_{gf} = K_i + U_{gi} \Rightarrow \frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$$

Using $v_f = v_c, y_f = \frac{1}{3}L, v_i = 0,$ and the above value for v_C^2, we get

$$\frac{1}{2}mg\frac{L}{3} + mg\frac{L}{3} = mgy_i \Rightarrow y_i = \frac{L}{2}$$

That is, the ball is a vertical distance $\frac{1}{2}L$ above the peg's location or a distance of

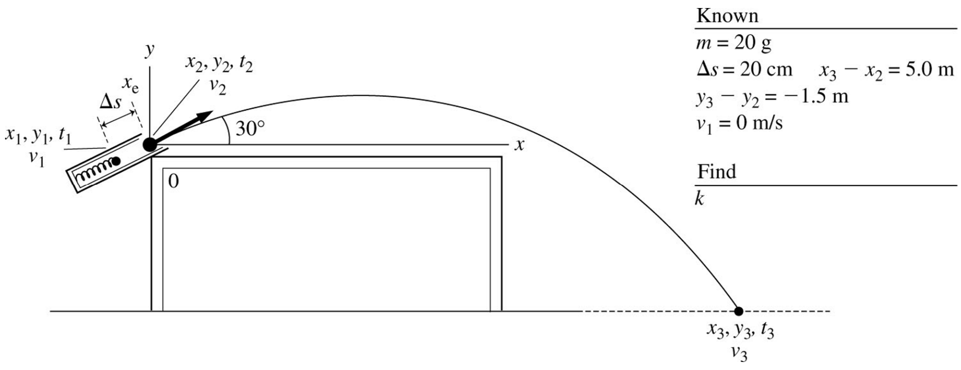
$$\left(\frac{2L}{3} - \frac{L}{2}\right) = \frac{L}{6}$$

below the point of suspension of the pendulum, as shown in the figure on the right. Thus,

$$\cos\theta = \frac{L/6}{L} = \frac{1}{6} \Rightarrow \theta = 80.4°$$

10.70. Model: Assume an ideal spring that obeys Hooke's law. The mechanical energy $K + U_\text{s} + U_\text{g}$ is conserved during the launch of the ball.

Visualize:



This is a two-part problem. In the first part, we use projectile equations to find the ball's velocity v_2 as it leaves the spring. This will yield the ball's kinetic energy as it leaves the spring.

Solve: Using the equations of kinematics,

$$x_3 = x_2 + v_{2x}(t_3 - t_2) + \frac{1}{2}a_x(t_3 - t_2)^2 \Rightarrow 5.0 \text{ m} = 0 \text{ m} + (v_2 \cos 30°)(t_3 - 0 \text{ s}) + 0 \text{ m}$$

$$(v_2 \cos 30°)t_3 = 5.0 \text{ m} \Rightarrow t_3 = (5.0 \text{ m}/v_2 \cos 30°)$$

$$y_3 = y_2 + v_{2y}(t_3 - t_2) + \frac{1}{2}a_y(t_3 - t_2)^2$$

$$-1.5 \text{ m} = 0 + (v_2 \sin 30°)(t_3 - 0 \text{ s}) + \frac{1}{2}(9.8 \text{ m/s}^2)(t_3 - 0 \text{ s})^2$$

Substituting the value for t_3, $(-1.5 \text{ m}) = (v_2 \sin 30°)\left(\dfrac{5.0 \text{ m}}{v_2 \cos 30°}\right) - (4.9 \text{ m/s}^2)\left(\dfrac{5.0 \text{ m}}{v_2 \cos 30°}\right)^2$

$$\Rightarrow (-1.5 \text{ m}) = +(2.887 \text{ m}) - \frac{163.33}{v_2^2} \Rightarrow v_2 = 6.102 \text{ m/s}$$

The conservation of energy equation $K_2 + U_{\text{s}2} + U_{\text{g}2} = K_1 + U_{\text{s}1} + U_{\text{g}1}$ is

$$\frac{1}{2}mv_2^2 + \frac{1}{2}k(0 \text{ m})^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}k(\Delta s)^2 + mgy_1$$

Using $y_2 = 0$ m, $v_1 = 0$ m/s, $\Delta s = 0.20$ m, and $y_1 = -(\Delta s)\sin g30°$, we get

$$\frac{1}{2}mv_2^2 + 0 \text{ J} + 0 \text{ J} = 0 \text{ J} + \frac{1}{2}k(\Delta s)^2 - mg(\Delta s)\sin 30° \quad (\Delta s)^2 k = mv_2^2 + 2mg(\Delta s)\sin 30°$$

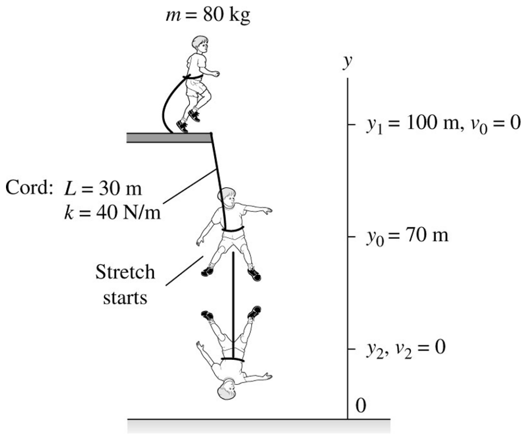
$$(0.20 \text{ m})^2 k = (0.020 \text{ kg})(6.102 \text{ m/s})^2 + 2(0.020 \text{ kg})(9.8 \text{ m/s}^2)(0.20)(0.5) \Rightarrow k = 19.6 \text{ N/m}$$

The final answer rounds to 20 N/m.

Assess: Note that $y_1 = -(\Delta s)\sin 30°$ is with a minus sign and hence the gravitational potential energy mgy_1 is $-mg(\Delta s)\sin 30°$.

10.71. Model: Choose yourself + spring + earth as the system. There are no forces from outside this system, so it is an isolated system. The interaction forces within the system are the spring force of the bungee cord and the gravitational force. These are both conservative forces, so mechanical energy is conserved.

Visualize:



We can equate the system's initial energy, as you step off the bridge, to its final energy when you reach the lowest point. We do *not* need to compute your speed at the point where the cord starts to stretch. We do, however, need to note that the end of the *unstretched* cord is at $y_0 = y_1 - 30 \text{ m} = 70 \text{ m}$, so $U_{2s} = \frac{1}{2}k(y_2 - y_0)^2$. Also note that $U_{1s} = 0$, since the cord is not stretched. The energy conservation equation is

$$K_2 + U_{2g} + U_{2s} = K_1 + U_{1g} + U_{1s} \Rightarrow 0 \text{ J} + mgy_2 + \frac{1}{2}k(y_2 - y_0)^2 = 0 \text{ J} + mgy_1 + 0 \text{ J}$$

Multiply out the square of the binomial and rearrange:

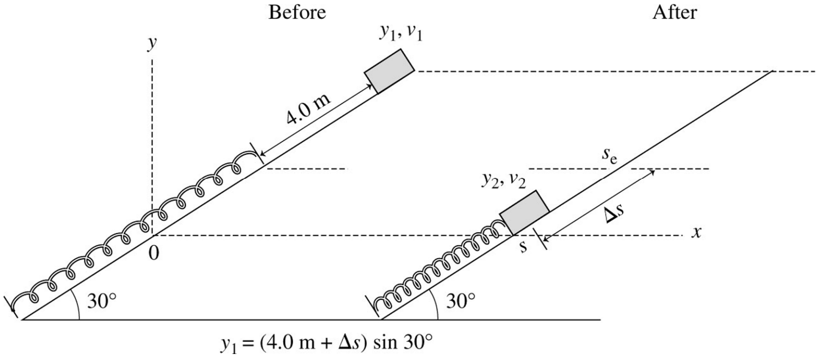
$$mgy_2 + \frac{1}{2}ky_2^2 - ky_0y_2 + \frac{1}{2}ky_0^2 = mgy_1$$

$$\Rightarrow y_2^2 + \left(\frac{2mg}{k} - 2y_0\right)y_2 + \left(y_0^2 - \frac{2mgy_1}{k}\right) = y_2^2 - 100.8y_2 + 980 = 0$$

This is a quadratic equation with roots 89.9 m and 10.9 m. The first is not physically meaningful because it is a height above the point where the cord started to stretch. So we find that your distance from the water when the bungee cord stops stretching is 10.9 m which is 11 m to two sig figs.

10.72. Model: Assume an ideal spring that obeys Hooke's law. There is no friction, hence the mechanical energy $K + U_g + U_s$ is conserved.

Visualize:



We have chosen to place the origin of the coordinate system at the point of maximum compression. We will use lengths along the ramp with the variable *s* rather than *x*.

Solve: **(a)** The conservation of energy equation $K_2 + U_{g2} + U_{s2} = K_1 + U_{g1} + U_{s1}$ is

$$\frac{1}{2}mv_2^2 + mgy_2 + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_1^2 + mgy_1 + k(0\ \text{m})^2$$

$$\frac{1}{2}m(0\ \text{m/s})^2 + mg(0\ \text{m}) + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}m(0\ \text{m/s})^2 + mg(4.0\ \text{m} + \Delta s)\sin 30° + 0\ \text{J}$$

$$\frac{1}{2}(250\ \text{N/m})(\Delta s)^2 = (10\ \text{kg})(9.8\ \text{m/s}^2)(4.0\ \text{m} + \Delta s)\left(\frac{1}{2}\right)$$

This gives the quadratic equation:

$$(125\ \text{N/m})(\Delta s)^2 - (49\ \text{kg} \cdot \text{m/s}^2)\Delta s - 196\ \text{kg} \cdot \text{m}^2/\text{s}^2 = 0$$
$$\Rightarrow \Delta s = 1.46\ \text{m and } -1.07\ \text{m (unphysical)}$$

The maximum compression is 1.46 m which rounds to 1.5 m.

(b) We will now apply the conservation of mechanical energy to a point where the vertical position is y and the block's velocity is v. We place the origin of our coordinate system on the free end of the spring when the spring is neither compressed nor stretched.

$$\frac{1}{2}mv^2 + mgy + \frac{1}{2}k(\Delta s)^2 = \frac{1}{2}mv_1^2 + mgy_1 + \frac{1}{2}k(0\ \text{m})^2$$

$$\frac{1}{2}mv^2 + mg(-\Delta s\ \sin 30°) + \frac{1}{2}k(\Delta s)^2 = 0\ \text{J} + mg(4.0\ \text{m}\ \sin 30°) + 0\ \text{J}$$

$$\frac{1}{2}k(\Delta s)^2 - (mg\sin 30°)\Delta s + \frac{1}{2}mv^2 - mg\sin 30°(4.0\ \text{m}) = 0$$

To find the compression where v is maximum, take the derivative of this equation with respect to Δs:

$$\frac{1}{2}k\ 2(\Delta s) - (mg\sin 30°) + \frac{1}{2}m\ 2v\frac{dv}{d\Delta s} - 0 = 0$$

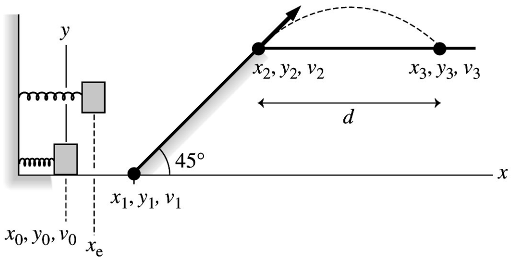
Since $\dfrac{dv}{d\Delta s} = 0$ at the maximum, we have

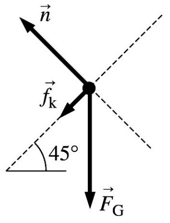
$$\Delta s = (mg\sin 30°)/k = (10\ \text{kg})(9.8\ \text{m/s}^2)(0.5)/(250\ \text{N/m}) = 19.6\ \text{cm}$$

This is 20 cm to two sig figs.

10.73. **Model:** Assume the spring to be ideal that obeys Hooke's law, and model the block as a particle.

Visualize: We place the origin of the coordinate system on the free end of the compressed spring which is in contact with the block. Because the horizontal surface at the bottom of the ramp is frictionless, the spring energy appears as kinetic energy of the block until the block begins to climb up the incline.





Solve: Although we could find the speed v_1 of the block as it leaves the spring, we don't need to. We can use energy conservation to relate the initial potential energy of the spring to the energy of the block as it begins projectile motion at point 2. However, friction requires us to calculate the increase in thermal energy. The energy equation is

$$K_2 + U_{g2} + \Delta E_{th} = K_0 + U_{g0} + W_{ext} \quad \Rightarrow \quad \tfrac{1}{2}mv_2^2 + mgy_2 + f_k\Delta s = \tfrac{1}{2}k(x_0 - x_e)^2$$

The distance along the slope is $\Delta s = y_2/\sin(45°)$. The friction force is $f_\text{k} = \mu_\text{k} n,$ and we can see from the free-body diagram that $n = mg\cos(45°)$. Thus

$$v_2 = \sqrt{\frac{k}{m}(x_0 - x_\text{e})^2 - 2gy_2 - 2\mu_\text{k} g y_2 \cot(45°)}$$

$$= \left[\frac{1000\text{ N/m}}{0.20\text{ kg}}(0.15\text{ m})^2 - 2(9.8\text{ m/s}^2)(2.0\text{ m}) - 2(0.20)(9.8\text{ m/s}^2)(2.0\text{ m})\cot(45°)\right]^{1/2} = 8.091\text{ m/s}$$

Having found the velocity v_2, we can now find $(x_3 - x_2) = d$ using the kinematic equations of projectile motion:

$$y_3 = y_2 + v_{2y}(t_3 - t_2) + \frac{1}{2}a_{2y}(t_3 - t_2)^2$$

$$2.0\text{ m} = 2.0\text{ m} + v_2\sin(45°)(t_3 - t_2) + \tfrac{1}{2}(-9.8\text{ m/s}^2)(t_3 - t_2)^2$$

$$t_3 - t_2 = 0\text{ s and } 1.168\text{ s}$$

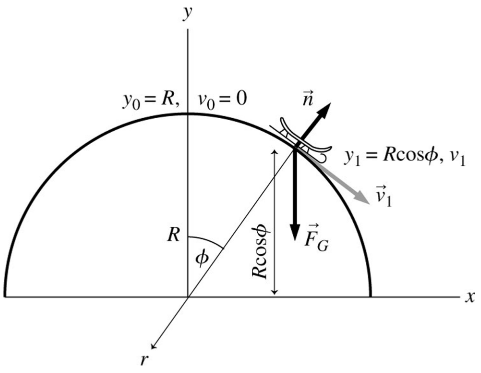
Finally,

$$x_3 = x_2 + v_{2x}(t_3 - t_2) + \frac{1}{2}a_{2x}(t_3 - t_2)^2$$

$$d = (x_3 - x_2) = v_2\cos(45°)(1.168\text{ s}) + 0\text{ m} = 6.7\text{ m}$$

10.74. Model: Model the sled as a particle. Because there is no friction, the sum of the kinetic and gravitational potential energy is conserved during motion.

Visualize:



Place the origin of the coordinate system at the center of the hemisphere. Then $y_0 = R$ and, from geometry, $y_1 = R\cos\phi.$

Solve: The energy conservation equation $K_1 + U_1 = K_0 + U_0$ is

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_0^2 + mgy_0 \Rightarrow \frac{1}{2}mv_1^2 + mgR\cos\phi = mgR \Rightarrow v_1 = \sqrt{2gR(1 - \cos\phi)}$$

(b) If the sled is on the hill, it is moving in a circle and the r-component of $\vec{F}_{\text{net}}$ has to point to the center with magnitude $F_{\text{net}} = mv^2/R$. Eventually the speed gets so large that there is not enough force to keep it in a circular trajectory, and that is the point where it flies off the hill. Consider the sled at angle ϕ. Establish an r-axis pointing toward the center of the circle, as we usually do for circular motion problems. Newton's second law along this axis requires:

$$(F_{\text{net}})_r = F_{\text{G}}\cos\phi - n = mg\cos\phi - n = ma_r = \frac{mv^2}{R}$$

$$\Rightarrow n = mg\cos\phi - \frac{mv^2}{R} = m\left(g\cos\phi - \frac{v^2}{R}\right)$$

The normal force decreases as v increases. But n can't be negative, so the fastest speed at which the sled stays on the hill is the speed v_{max} that makes $n \to 0$. We can see that $v_{\text{max}} = \sqrt{gR\cos\phi}$.

(c) We now know the sled's speed at angle ϕ, and we know the maximum speed it can have while remaining on the hill. The angle at which v reaches v_{max} is the angle ϕ_{max} at which the sled will fly off the hill. Combining the two expressions for v_1 and v_{max} gives:

$$\sqrt{2gR(1-\cos\phi)} = \sqrt{gR\cos\phi} \Rightarrow 2R(1-\cos\phi_{\text{max}}) = R\cos\phi_{\text{max}}$$

$$\Rightarrow \cos\phi_{\text{max}} = \frac{2}{3} \Rightarrow \phi_{\text{max}} = \cos^{-1}\left(\frac{2}{3}\right) = 48°$$